Final Exam

Math 20F
8/22/08

Name: ________________________________
Section: ________________________________

Read all of the following information before starting the exam:

- READ EACH OF THE PROBLEMS OF THE EXAM CAREFULLY!
- Show all work, clearly and in order, if you want to get full credit. I reserve the right to take off points if I cannot see how you arrived at your answer (even if your final answer is correct).
- A single 8 1/2 $\times$ 11 sheet of notes (double sided) is allowed. No calculators are permitted.
- Circle or otherwise indicate your final answers.
- Please keep your written answers clear, concise and to the point.
- This test has 5 problems and is worth ... points. It is your responsibility to make sure that you have all of the pages!
- Turn off cellphones, etc.
- Good luck!

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1. (*0 points*)

(**a**) Let $\mathcal{D} = \{f(x) \in \mathbb{P}_n : f'(0) = 0\}$ denote the set of functions in $\mathbb{P}_n$ whose derivative at zero is 0. Verify that $\mathcal{D}$ is a subspace of $\mathbb{P}_n$.

(**b**) Suppose $A = \begin{pmatrix} 2 & -1 & 0 & 3 \\ -1 & 0 & 3 & 2 \end{pmatrix}$. Find a basis for $\mathrm{Col}(A)$ and $\mathrm{Nul}(A)$. What is $\mathrm{rank}(A)$?.

(c) Suppose $A = m \times n$ with $m < n$. Suppose $\text{rank}(A) < n$. Is it possible that the columns of A span $\mathbb{R}^m$? Why or why not?

(d) Suppose $H = \left\{ \begin{pmatrix} a + 2b + 3c \\ a + 2b + 3c \\ a + 2b + 3c \end{pmatrix} : a, b, c \in \mathbb{R} \right\}$. Find a basis for H.

2. (*0 points*) **(a)** Suppose $A = \begin{pmatrix} 3 & -3 & 5 \\ 0 & 4 & -3 \\ 0 & 2 & -1 \end{pmatrix}$. Find the eigenvalues of A with multiplicity.

(b) Diagonalize the matrix A from part (1).

(c) Suppose A has eigenvalues 2 and 3 with corresponding eigenvalues $\mathbf{x}$ and $\mathbf{y}$ respectively. Suppose $\mathbf{z} = 10\mathbf{x} + 2\mathbf{y}$. Compute $A^{100}\mathbf{z}$. You may leave your answers in terms of $\mathbf{x}$ and $\mathbf{y}$.

3. (*0 points*) (**a**) Use the Gramm-Schmitt process to make $\mathcal{B} = \left\{ \begin{pmatrix} 1 \\ -1 \end{pmatrix}, \begin{pmatrix} 2 \\ 3 \end{pmatrix} \right\}$ an orthogonal basis for $\mathbb{R}^2$.

(**b**) Find a QR factorization for

$$A = \begin{pmatrix} 1 & 0 & -1 \\ 1 & -1 & -1 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{pmatrix}.$$

(**c**) For the matrix A in part (2) find the least squares solution to $A\mathbf{x} = \begin{pmatrix} 1 \\ 2 \\ -1 \\ 7 \end{pmatrix}$.

4. (*0 points*) For each statement, mark it true of false. If it is false give a (counter)example. If it is true give a reason - if the reason is a theorem, state the theorem, otherwise give a brief proof. No credit for answers without a correct reason or example. Unless explicitly stated, no assumptions are made on the dimensions of matrices.

(a) If A has n different eigenvectors, then A is diagonalizable.

(b) If $AP = PD$, where D is diagonal, the columns of P are eigenvectors of A.

(c) If λ is an eigenvalue of A, λ^{100} is an eigenvalue of A^{100}.

(d) If $\mathcal{B}$ and $\mathcal{C}$ are two bases of $\mathbb{R}^n$, the change of basis matrix from $\mathcal{B}$ to $\mathcal{C}$ is invertible.

(e) An orthogonal matrix has orthonormal rows.

(f) If AB is invertible, and A and B are square, then A is invertible.

(g) If $\hat{\mathbf{x}}$ is the least squares solution to $A\mathbf{x} = \mathbf{b}$, then $\hat{\mathbf{b}} = A\hat{\mathbf{x}}$ is the closest vector in $\mathrm{Col}(A)$ to $\mathbf{b}$.

(h) If A is diagonalizable, $\det(A)$ is the product of the eigenvalues of A.

5. (*0 points*) (**a**) Suppose $A^{-1} = \begin{pmatrix} 1 & 2 & -1 \\ 0 & -1 & 2 \\ 2 & 3 & 3 \end{pmatrix}$. Solve $A\mathbf{x} = \mathbf{b}$.

(**b**) Define eigenvalue.

(**c**) Suppose $T : \mathbb{R}^2 \to \mathbb{R}^2$ is a linear transformation that reflects across the x_1-axis and then through the line $x_1 = x_2$. Find the matrix for T.

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