Random Variables

Discrete

p.m.f. $\boxed{P_X(k) = P(X=k)}$

(probability mass function)

a) $0 \le P_X(k) \le 1$

b) $\sum_{\substack{\text{all} \\ \text{possible} \\ \text{values } k}} P_X(k) = 1$

Continuous

p.d.f $f_X(x)$

(probability density function)

a) $f_X(x) \ge 0$

b) $\int_{-\infty}^{\infty} f_X(x)\,dx = 1$

$$\boxed{P(a \le X \le b) = \int_a^b f_X(x)\,dx}$$

$$P(X=a) = 0$$

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both for discrete and/or continuous

$$\boxed{\text{c.d.f. } F_X(x) = P(X \le x)}$$

(cumulative distribution function / same definition for discrete or continuous)

Properties:

① $0 \le F_X(x) \le 1$

② $F_X(-\infty) = 0$, $F_X(+\infty) = 1$

③ $F_X(x)$ is continuous from the right

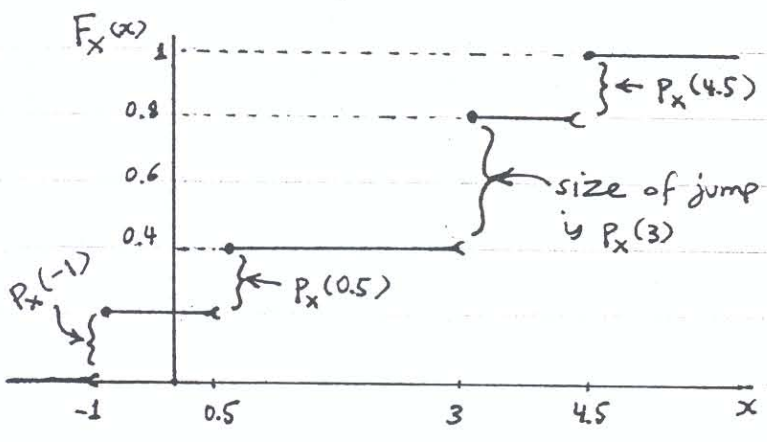
i.e. $F_X(x+\varepsilon) \to F_X(x)$ as $\varepsilon \downarrow 0$

④ $F_X(x)$ is non-decreasing

Also: $\boxed{P(a < X \le b) = F_X(b) - F_X(a)}$

DISCRETE

$F_X(x)$ is ladder-like with jumps of size $P_X(k)$ at the possible k-values



CONTINUOUS

$F_X(x)$ is continuous function and $\boxed{f_X(x) = F_X'(x)}$