JOINT DISTRIBUTIONS

Definition c.d.f. $F_{XY}(x,y) = P(X \le x \underset{\uparrow\,\text{and (i.e. } \cap)}{,} Y \le y)$

X and Y are independent $\Leftrightarrow$ $\overset{\text{joint c.d.f.}}{F_{XY}(x,y)} = \overset{\text{marginal c.d.f.'s}}{F_X(x)\, F_Y(y)}$ for all x, y.

DISCRETE R.V.s

joint p.m.f.

$$p_{XY}(x,y) = P(X = x \underset{\uparrow\,\text{and}}{,} Y = y)$$

"marginal" (i.e. individual) p.m.f

$$p_X(x) = \sum_y p_{XY}(x,y)$$

$$p_Y(y) = \sum_x p_{XY}(x,y)$$

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conditional p.m.f.

$$p_{X|Y}(x|y) = \frac{p_{XY}(x,y)}{p_Y(y)}$$

$$p_{Y|X}(y|x) = \frac{p_{XY}(x,y)}{p_X(x)}$$

—·—

X and Y are independent

$$\Updownarrow \quad \text{for all } x,y$$

$$\underset{\text{joint p.m.f.}}{p_{XY}(x,y)} = \underset{\text{marginal}}{p_X(x) \cdot p_Y(y)}$$

CONTINUOUS R.V.s

joint p.d.f $f_{XY}(x,y) = \dfrac{\partial^2}{\partial x \partial y} F_{XY}(x,y)$

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DEFINING PROPERTY OF JOINT P.D.F.:

$$P\left(\begin{bmatrix} X \\ Y \end{bmatrix} \in A\right) = \iint_A f_{XY}(x,y)\, dx\, dy$$

where A is a subset of $\mathbb{R}^2$.

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"marginal" (i.e. individual) p.d.f.

$$f_X(x) = \int_{-\infty}^{\infty} f_{XY}(x,y)\, dy$$

$$f_Y(y) = \int_{-\infty}^{\infty} f_{XY}(x,y)\, dx$$

—·—

conditional p.d.f.

$$f_{X|Y}(x|y) = \frac{f_{XY}(x,y)}{f_Y(y)}$$

$$f_{Y|X}(y|x) = \frac{f_{XY}(x,y)}{f_X(x)}$$

—·—

X and Y are independent

$$\Updownarrow$$

$$\underset{\uparrow\,\text{joint p.d.f.}}{f_{XY}(x,y)} = \underset{\uparrow\,\text{marginal p.d.f.'s}}{f_X(x) \cdot f_Y(y)} \quad \text{for all } x,$$