287C HW (due Wed April 30 in class)

1. Find the BLUE estimator of the mean of an AR(1) time stationary series, and compare it to the sample mean asymptotically. [HINT: to get an approximation to the inverse covariance matrix, you may construct the Toeplitz matrix from the inverse autocovariance sequence]

2. Use some statistical software (S+, R, etc.) to generate data of sample size n from an AR(1) and an MA(1) with different parameter values. Plot the estimated ACF for lag up to n and note the regions where the estimate is reliable. Try n=100,500 and 1000.

3. Let $\Phi_X(w)$ be the FFT of data $X_1, ..., X_n$, i.e., let $\Phi_X(w) = (2\pi n)^{-1/2} \sum_{t=1}^{n} X_t e^{itw}$, and let $I_X(w)$ be the periododogram, i.e., $I_X(w) = |\Phi_X(w)|^2$. Assume for simplicity the MA(1) model: $X_t = Z_t + \theta_1 Z_{t-1}$ where $Z_t \sim$ i.i.d. $(0,\sigma^2)$. [A similar argument holds for MA(q), and by MA(q) approximation, for any linear time series.]

a. Let $\theta(z) = 1+\theta_1 z$, and show that $\Phi_X(w) = \theta(e^{iw})\Phi_Z(w)+R_n(w)$ where, as $n \to \infty$, $R_n(w)$ converges to zero in probability uniformly in w, i.e., $R_n(w) = o_P(1)$ where the $o_P(1)$ does not depend on w.

b. Assume $Z_t^4 < \infty$ and use part (a) to show that $I_X(w) = |\theta(e^{iw})|^2 I_Z(w)+r_n(w)$ where $r_n(w) = o_P(1)$.

c. Assuming $Z_t^4 < \infty$, the assertion of part (b) can be strengthened to $Er_n^2(w) = o(1)$, i.e., $r_n(w)$ converges to zero in Mean Square. Use the strengthened result to show that $I_X(w_j)$ and $I_X(w_k)$ are asymptotically uncorrelated if $j \neq k$ and $w_k = 2\pi k/n$. [Hint: $I_Z(w_j)$ and $I_Z(w_k)$ are asymptotically uncorrelated due to a trigonometric identity –see Proposition 10.1.1 of Brockwell and Davis.]

4. Do ex. 1 and ex. 5 from Ch. 7 of Brockwell and Davis. [For ex. 1, proving asymptotic normality means verifying the conditions of the respective theorem in Ch. 7].