Time-varying NoVaS vs. GARCH: robustness against structural breaks in financial returns

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Consider financial returns data $Y_1, \ldots, Y_n$ satisfying $EY_t = 0$ for all t. The NoVaS transformation of Politis (2007) is defined as

$$W_{t,a} = \frac{Y_t}{\sqrt{\alpha s_{t-1}^2 + a_0 Y_t^2 + \sum_{i=1}^p a_i Y_{t-i}^2}} \quad \text{for} \quad t = p+1, p+2, \ldots, n \qquad (1)$$

where $s_{t-1}^2 = (t-1)^{-1} \sum_{k=1}^{t-1} Y_k^2$, and the coefficients $\alpha, a_0, a_1, \ldots, a_p$ (and the order p) are selected such that the series $\{W_{t,a}$ for $t = p+1, p+2, \ldots\}$ is (approximately) i.i.d. $N(0,1)$. The transformation can be inverted to yield

$$Y_t = \frac{W_{t,a}}{\sqrt{1 - a_0 W_{t,a}^2}} \sqrt{\alpha s_{t-1}^2 + \sum_{i=1}^p a_i Y_{t-i}^2} \ \text{ for } \ t = p+1, \ldots, n. \qquad (2)$$

Eq. (2) is a model-like equation that can be used in lieu of a model for prediction. In fact, the NoVaS transformation is just an application of the Model-free Prediction Principle of Politis (2015) to financial returns data.

Financial returns are often assumed to be strictly stationary. Nevertheless, if the data $Y_1, \ldots, Y_n$ span a long time interval, e.g. daily financial returns spanning several years, it may be unrealistic to assume that the stochastic structure of time series $\{Y_t, t \in \mathbf{Z}\}$ has stayed invariant over such a long stretch of time. Instead, one can assume a slowly-changing stochastic structure, i.e., a locally stationary model. Indeed, the theory of time–varying ARCH (TV-ARCH) processes was developed to capture such a phenomenon; see Dahlhaus and Subba Rao (2006).

Let $g(\cdot)$ denote a function of interest; in order to predict $g(Y_{t+1})$ based on data $\{Y_s,\ s \le t\}$ via a TV-GARCH(1,1) model, we can simply fit a standard GARCH(1,1) model using as data the subseries $Y_{t-b+1}, \ldots, Y_t$. Here, the window size b should be large enough so that accurate estimation of the GARCH parameters is possible based on the subseries $Y_{t-b+1}, \ldots, Y_t$ but small enough so that such a subseries can plausibly be considered stationary.

In a similar vein, we can predict $g(Y_{t+1})$ by fitting one of the NoVaS algorithms (Simple vs. Exponential, etc.) just using the 'windowed' data $Y_{t-b+1}, \ldots, Y_t$. In so doing, we are constructing a **time–varying NoVaS** (TV-NoVaS) transformation. In numerical work, Politis and Thomakos (2012) showed that NoVaS fitting can be done more efficiently than GARCH fitting by (numerical) MLE. Thus, it is expected that TV-NoVaS may be able to capture a changing stochastic structure in a more flexible manner.

We confirm this conjecture via simulation with data from a TV-GARCH model. Note, however, that an alternative form of nonstationarity may be due to the possible presence of structural breaks, i.e., change points. Mikosch and Starica (2004) show the interesting effects that an undetected change point may have on our interpretation and analysis of ARCH/GARCH modeling. Hence, in our simulation we also include a structural break model, and indeed witness the instability of TV-GARCH predictors. By contrast, TV-NoVaS predictors appear robust, adapting effortlessly to the new regime; see Politis (2015) for more details.

Keywords: Financial returns, local stationarity, prediction.

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