Correction to “Banded and tapered estimates of autocovariance matrices and the linear process bootstrap”, J. Time Ser. Anal., vol. 31, pp. 471-482, 2010

Timothy L. McMurry
University of Virginia

Dimitris N. Politis*
University of California, San Diego

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In this note, we correct the proof and statement of Theorem 5 in McMurry and Politis (2010), which establishes the consistency of the linear process bootstrap (LPB) for the sample mean. The statement of Lemma 6, on which Theorem 5 relies, is in error.

Lemma 6 is used to bound the operator norm $\rho(A^{1/2} - B^{1/2})$ by a bounded factor times the operator norm $\rho(A-B)$, where $A^{1/2}$ and $B^{1/2}$ are taken to be the lower triangular Cholesky factors of A and B. The result was erroneously thought to be an extension of results in Horn and Johnson (1990) for the matrix square root given by the spectral decomposition. This lemma is used in several places throughout the proof of Theorem 5.[1]

The correct bound is slightly weaker than stated and can be found in Theorem 2.1 of Drmač, Omladič, and Veselić (1994). It is

$$\rho\left(A^{1/2} - B^{1/2}\right) \leq \rho(B^{1/2}) \frac{2c_n \rho\left(B^{-1/2}\right)^2 \rho\left(A - B\right)}{1 + \sqrt{1 - 4c_n^2 \rho\left(B^{-1/2}(A-B)(B^{-1/2})^t\right)}}, \tag{1}$$

where $c_n = 1/2 + \lfloor \log_2 n \rfloor$. In order to ensure the operator norm convergence of the Cholesky factors using (1), we now require conditions strong enough to guarantee $c_n^2 \rho(A-B) \to 0$; the conditions which ensure this rate of convergence are implied by all typical cases including those discussed in Corollary 4. With these modifications, Theorem 5 is correctly stated as follows.

Theorem 5. *Assume the conditions of Theorems 1, 2, and 3, with $q = 2$. Furthermore, assume $\beta > 1/2$ as in Corollary 2 and $[\log n]^2 \rho(\hat{\Sigma}_{\kappa,l} - \Sigma_n) = o_P(1)$. Let $E[X_i] = \mu$. Then,*

$$\sup_x \left| \mathrm{P}\left[n^{1/2}(\bar{X} - \mu) \leq x\right] - \mathrm{P}^*\left[n^{1/2}\bar{Y}^* \leq x\right] \right| \to_P 0, \tag{2}$$

and

$$\mathrm{var}^*\left[n^{1/2}\bar{Y}^*\right] \to_P \sigma^2,$$

where $\sigma^2 = (\gamma_0 + 2\sum_{k=1}^{\infty} \gamma_k) = \lim_{n\to\infty} \mathrm{var}\left[n^{1/2}\bar{X}\right]$.

The proof of Theorem 5 follows as published in McMurry and Politis (2010) with (1) substituted for all invocations of Lemma 6.

*dpolitis@ucsd.edu

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References

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