

Preface to the Murray Rosenblatt Memorial special issue of JTSA

Richard C. Bradley

Department of Mathematics, Indiana University, Bloomington, IN 47405, U.S.A.
bradleyr@indiana.edu

Richard A. Davis

Department of Statistics, Columbia University, New York, NY 10027, U.S.A.
rdavis@stat.columbia.edu

Dimitris N. Politis

Department of Mathematics and Halicioglu Data Science Institute,
University of California at San Diego, La Jolla, CA 92093, U.S.A.
dpolitis@ucsd.edu

This special issue of the *Journal of Time Series Analysis* is dedicated to the memory of Murray Rosenblatt (1926-2019). Murray, of course, was one of the giants in time series analysis who made fundamental and lasting contributions to the subject. His 1957 book *Statistical Analysis of Stationary Time Series* with Ulf Grenander became the standard reference in the subject for nearly three decades. It still contains many gems and continues to be a useful historical reference.

Notably, Murray's research contributions went beyond time series analysis, and include fundamental work that spans both probability and statistics. It is not possible here to begin to do justice to all of the contributions that he made to those fields. This Preface here will have a more modest goal: to briefly trace out some of the main strands in his research and career, and in particular to give some indication of the role of the field of time series analysis in providing either the setting or the inspiration for a considerable portion of his research. Some of the information below is taken from Boggs and Ni (2019), Bradley and Davis (2019), and Sun (1997).

Murray Rosenblatt was born on September 7, 1926. He earned a Bachelor of Science degree from the City College of New York in 1946, and then earned a Ph.D. degree in Mathematics at Cornell University in 1949. His Ph.D. dissertation, written under the direction of Mark Kac, was titled "On distributions of certain Wiener functionals" and was related to the Feynman-Kac formula. In the following year he married Adylin (Ady) Lipson and spent one more year at Cornell, as a postdoctoral fellow. After that, he held positions at the University of Chicago, Columbia University, Indiana University, and Brown University. Then in 1964 he took a position in the Mathematics Department at the newly created University of California at San Diego (UCSD), where he would spend the rest of his career, retiring in 1994.

Murray was highly recognized for his research. He was a Fellow of the American Mathematical Society (AMS), a Fellow of the Institute of Mathematical Statistics (IMS), and a Fellow of the Society for Industrial and Applied Mathematics (SIAM). He was awarded a Guggenheim Fellowship in 1964 and 1972. He delivered the IMS Wald Lectures in 1970 and was elected to the National Academy of Sciences in 1984. In 1997 an issue of the *Journal of Theoretical Probability* was published in honor of his 70th birthday. A volume of *Celebratio Mathematica* (celebratio.org), an online archive of mathematical people compiled and maintained as a public-interest project by Mathematical Sciences Publishers (msp.org), is currently being put together celebrating Murray's illustrious career.

In October 2016, a conference was held in honor of his 90th birthday; it was co-organized by Ruth Williams and other faculty members of the UCSD Mathematics Department. The 2016 conference also served to launch the Murray and Adylin Rosenblatt Endowed Lecture Series in Applied Mathematics; this is an annual series of lectures designed to bring prominent mathematicians and statisticians to UCSD to talk about the state-of-the-art in their field. Notably, in addition to his deep theoretical work on probability and statistics, Murray was also directly involved with applications of time series analysis in the study of real world data. For example, the paper of Helland, Lii, and Rosenblatt (1991), as well as two earlier joint papers by the same three authors, dealt with estimation problems in the study of turbulence. Hence, it is no wonder that Murray specifically requested that the Endowed Lecture Series bearing his name have an emphasis towards real world applications.

Murray passed away on October 9, 2019. His wife Ady preceded him in death ten years earlier, in 2009, after a courageous fight with cancer. They are survived by their two children: Karin and Daniel.

A total of 22 students earned their Ph.D.'s under Murray's direction, including 14 at UCSD, among whom two (RB and RD) are guest editors of this special issue. He always gave his students tremendous encouragement and support, which lasted well beyond their graduate school days. Together, he and his wife Ady served as surrogate parents to many of his students.

A few particularly influential papers. During his career, Murray published five books and more than 150 papers. A selection of about 35 of his most influential papers were reprinted in Davis et al. (2011). Furthermore, several of his papers became starting points for lines of research that remain active to this day. Here we shall mention a few particularly well known such papers that offered incredible foresight.

His paper “A central limit theorem and a strong mixing condition” (Rosenblatt, 1956a) introduced the “strong mixing” (alpha-mixing) condition, and gave a central limit theorem under that condition. As Richard Olshen commented in the late 1970s, part of the motivation for that paper was the intent to extend the methods of statistical inference in time series analysis to random processes that did not satisfy the usual “structural” assumptions such as in Markov chains, Gaussian processes, and the classical time series models. In the years following that paper, various other similar mixing conditions were introduced and developed by various researchers; and the research on limit theory under such conditions, and on the “structure” of random processes that satisfy such conditions, is still very active to this day (see e.g. Bradley, 2007). Nevertheless, strong mixing is still thought of as the “go to” mixing condition today.

In the same year, Murray's paper “Remarks on some nonparametric estimates of a density function” (Rosenblatt, 1956b) introduced kernel-type estimators of probability density, and provided some ideas behind the selection of optimal bandwidth for minimizing mean square error. That paper has inspired much further research on kernel-type estimation of probability density, not only in the context of independent, identically distributed random observations but also in the broader context of data from strictly stationary sequences such as ones that satisfy “mixing-type” dependence conditions.

His paper “Independence and Dependence” (Rosenblatt, 1961) provided a simple example (a particular “instantaneous function” of a stationary Gaussian sequence whose covariances were positive and decayed at a slow, non-summable rate) which had some “nice” moment properties but for which the partial sums failed to be asymptotically normal. Murray identified the limiting distribution for the (suitably normalized) partial sums; that limiting distribution later became known as the Rosenblatt distribution. That paper inspired research by other people on related “long-memory” random processes, and resulting limiting processes have become known as Rosenblatt processes (see e.g. Taqqu, 2011). This 1961 paper of Murray illustrates the abiding interest that he had in (counter)examples, which he found useful for illuminating the limits of statements of certain results and more generally, discovering insights into the “bigger picture of what is going on”.

His 1952 paper entitled “Remarks on a multivariate transformation” delineated a transformation that can map a random vector to a vector whose entries are independent and uniformly distributed on $[0,1]$. It is now known as the Rosenblatt transformation, and it is found useful in the study of copulas with applications to goodness-of-fit tests for multivariate distributions; see e.g. Nelsen (2006) or Genest et al. (2009).

Another line of Murray’s research has antecedents in Lévy’s work on random rotations on a circle, and in work done in the 1950s by researchers such as J.G. Wendel and K. Urbanik on probability measures, and limit theorems for them, on compact topological groups. A series of three papers of Murray’s (one joint with M. Heble) in the 1960s gave some further development of the theory of limits of convolutions of probability measures on compact topological semigroups. The third of those three papers, Rosenblatt (1965), applied that theory to products of random square matrices. For a brief account of those three papers, and of later work by other researchers inspired by them, see Sun (1997).

Murray’s books on Time Series Analysis and Markov Processes. A major theme throughout Murray’s career, already touched on above, was the role that the field of time series analysis played as the setting and/or the motivation for much of his research. The comments given below can provide only a further limited look at that theme.

Of the five books that Murray published, three directly involved the field of time series analysis. His first book, Grenander and Rosenblatt (1957), alluded to above, is considered a classic to this day.

Next, the book of Rosenblatt (1985) devoted much space to the estimation of covariances, cumulants, spectral density, and higher order spectra for random fields under certain strong mixing conditions. Murray’s interest in the estimation of the spectral density and higher order spectra, and in other uses of Fourier techniques in the analysis of time series, was an ongoing theme in his research throughout his career.

Murray’s final book, Rosenblatt (2000), dealt in part with a theme that played a role in much of his research later in his career: that in the analysis of time series from non-Gaussian phenomena, one could take advantage of some non-Gaussian features to perform tasks that are infeasible in the Gaussian data setting; an example is checking a time series for causality and/or invertibility.

Another theme of particular interest to Murray was Markov chains and Markov processes. That was the topic of many of his papers, as well as an earlier book of his, namely Rosenblatt (1971).

The final chapter of the 1971 book was devoted to the study of mixing conditions for Markov chains. In particular, there (and in a paper of his a year earlier) a central limit theorem (CLT) was presented for “instantaneous functions” of strictly stationary Markov chains that satisfy the ρ -mixing condition (which he identified under different terminology). In that CLT, the only moment assumptions were finite second moments and the growth of the variances of the partial sums. That seems to have been the first known explicit use of the ρ -mixing condition in a central limit theorem – although the ρ -mixing condition itself had already been present in other types of results since that condition was introduced by Kolmogorov and Rozanov (1960). A few years later, Ibragimov (1975) developed central limit theorems under a broader class of strictly stationary (not necessarily Markovian) ρ -mixing random sequences; the ρ -mixing condition has played a prominent role in much limit theory developed since then by many researchers.

The final chapter of Murray’s 1971 book also looked at some “structural” questions involving mixing conditions for Markov chains. In particular, Murray gave a peculiar class of examples of strictly stationary, ρ -mixing Markov chains for which, for every positive integer n , the n -step transition distributions are almost surely totally singular with respect to the (invariant) marginal distribution. In that chapter, he also examined the use of operator-theoretic techniques, in particular the Riesz Interpolation Theorem, for the comparison of various dependence conditions for Markov chains; those ideas found uses later on in broader (not necessarily Markovian) contexts by other researchers in the development of covariance inequalities and related results as well as in the comparison of certain “mixing-type” dependence conditions.

Some final comments. Murray had an uncanny insight into time series and stochastic processes in general. He had a nose for asking insightful questions and formulating interesting problems. For example, a conjecture of Wiener (1958) on the structure of stationary processes seemed to inspire Murray in a number of research directions. Wiener’s conjecture was that for a given strictly stationary sequence $(X_k, k \in \mathbb{Z})$ to have a representation of the form $X_k = f(\zeta_k, \zeta_{k-1}, \zeta_{k-2}, \dots)$ where $(\zeta_k, k \in \mathbb{Z})$ is an i.i.d. sequence and $f : \mathbb{R}^{\mathbb{N}} \rightarrow \mathbb{R}$ is a Borel function, it is necessary and sufficient that the backward tail σ -field of the sequence $(X_k, k \in \mathbb{Z})$ be trivial. In Rosenblatt (2009), Murray pointed out a counterexample, showing that the latter condition is not sufficient for such a representation. Even though the conjecture turned out to be false, the impetus of that work of Wiener can be found in Murray’s last papers.

Even in failing health, Murray attended sessions at the long running NBER/NSF—sponsored time series workshop held at UCSD in 2018. Even the leading statisticians and econometricians in attendance were in awe of Murray’s presence. This was a person who truly laid the groundwork on which much of time series analysis has been built. The famous quote by Isaac Newton, “If I have seen further than others, it is by standing upon the shoulders of giants.”, never seemed more apt.

For the Murray Rosenblatt Memorial special issue of JTSA, a number of notable time series researchers have agreed to contribute their work as a tribute to one of the giants of our field. The topics span classical results with a modern twist, such as autoregressive approximations of nonlinear processes, but also uniquely 21st century subject material, such as sparsity and high-dimensional models. We are hoping that the readers will appreciate the influence of Murray’s work on the papers of the special issue, and on Time Series Analysis in general.

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