**Problem 1.** Show that
\[
A = \begin{bmatrix}
0 & 1 & 6 & 1 \\
-1 & -1 & 1 & 0 \\
1 & 2 & 5 & 1 \\
\end{bmatrix}
\]
is row equivalent to
\[
\begin{bmatrix}
1 & 0 & -7 & -1 \\
0 & 1 & 6 & 1 \\
0 & 0 & 0 & 0 \\
\end{bmatrix}
\]
using the Row Reduction Algorithm.

You must show all your calculations and indicate what row operation you are using at each stage!

**Problem 2.** Find the general solutions of the systems whose augmented matrices are given below. Indicate all the basic and free variables and show all the row reduction calculations.

(a.)
\[
\begin{bmatrix}
1 & 1 & | & 1 \\
-4 & -3 & | & -2 \\
3 & 2 & | & 1 \\
\end{bmatrix}
\]
(b.)
\[
\begin{bmatrix}
1 & -2 & 1 & | & 3 \\
3 & -6 & -2 & | & 2 \\
\end{bmatrix}
\]

**Problem 3.** For each of the following system of linear equations, write the augmented matrix and use it to find all solutions to the following system. Indicate all the basic and free variables.

You may use calculator or computer to help with the Row Reduction Algorithm but also make sure you can do the steps by hand to prepare for the exams.

a. 
\[
\begin{align*}
2x_1 & - 3x_2 + x_3 + 7x_4 = 14 \\
2x_1 & + 8x_2 - 4x_3 + 5x_4 = -1 \\
x_1 & + 3x_2 - 3x_3 = 4 \\
-5x_1 & + 2x_2 + 3x_3 + 4x_4 = -19 \\
\end{align*}
\]

b. 
\[
\begin{align*}
3x_1 & + 4x_2 - x_3 + 2x_4 = 6 \\
x_1 & - 2x_2 + 3x_3 + x_4 = 2 \\
10x_2 & - 10x_3 - x_4 = 1 \\
\end{align*}
\]

c. 
\[
\begin{align*}
2x_1 & + 4x_2 + 5x_3 + 7x_4 = -26 \\
x_1 & + 2x_2 + x_3 - x_4 = -4 \\
-2x_1 & - 4x_2 + x_3 + 11x_4 = -10 \\
\end{align*}
\]

**Problem 4.** Let \( v_1 = \begin{bmatrix} 2 \\ -1 \\ 3 \\ 4 \end{bmatrix} \), \( v_2 = \begin{bmatrix} 3 \\ 2 \\ -2 \\ -1 \end{bmatrix} \), and \( W = \text{Span}\{v_1, v_2\} \). For each of the vector \( u \) below, determine if \( u \in W \). In the case that \( u \in W \), also provide an explicit linear combination that
demonstrates this (i.e. find the scalars $\alpha, \beta$ such that $\mathbf{u} = \alpha \mathbf{v}_1 + \beta \mathbf{v}_2$).

\[ (a.) \quad \mathbf{u} = \begin{bmatrix} 5 \\ 8 \\ -12 \\ -5 \end{bmatrix}, \quad (b.) \quad \mathbf{u} = \begin{bmatrix} 5 \\ 1 \\ 3 \\ 5 \end{bmatrix}, \quad (c.) \quad \mathbf{u} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}. \]

In addition, for part (c.), can you find the non-zero scalars $\alpha, \beta$ that make $\mathbf{u} = \alpha \mathbf{v}_1 + \beta \mathbf{v}_2$?

**Problem 5.** Consider the following system of linear equations

\[
\begin{align*}
    x_1 + x_2 + hx_3 &= k \\
x_1 + x_3 &= 3 \\
x_2 + 2x_3 &= -2.
\end{align*}
\]

Find all values of $h$ and $k$ such that the given system has (a.) no solution, (b.) a unique solution, and (c.) infinitely many solutions.

**Problem 6.** Answer the following statements with True or False. If true, give the approximate location in the textbook where a similar statement appears, or refer to a definition or theorem. If false, give a counter-example to show that the statement is not true for all cases.

a. Performing the elementary row operations on an augmented matrix will not change the solution set of the associated linear system.

b. Two matrices are row equivalent if they have the same number of rows.

c. The echelon form of a matrix is unique.

d. If the coefficient matrix for a system is a $3 \times 5$ with three pivot columns, then the system is consistent.

e. If the augmented matrix for a system is a $3 \times 5$ with the fifth column being a pivot column, then the system is consistent.

f. The weights $c_1, \ldots, c_p$ in a linear combination $c_1 \mathbf{v}_1 + \cdots + c_p \mathbf{v}_p$ cannot all be zero.

g. The vector $\frac{1}{2} \mathbf{v}_2$ is a linear combination of vectors $\mathbf{v}_1$ and $\mathbf{v}_2$.

h. The columns of a $2 \times 3$ matrix always span $\mathbb{R}^2$.

i. If $\mathbf{u}$ and $\mathbf{v}$ are nonzero vectors, then $\text{Span}\{\mathbf{u}, \mathbf{v}\}$ contains a line through $\mathbf{u}$ and the origin.

j. For any two vectors $\mathbf{u}$ and $\mathbf{v}$, the set $\text{Span}\{\mathbf{u}, \mathbf{v}\}$ is always visualized as a plan through the origin.

**Problem 7.** Set up a system of equations and use it to find all three-digit numbers satisfying the two following properties:

1. The tens-digit and the ones-digit add up to 5.

2. If the number is written with the digits in the reverse order, and then subtracted from the original number, the result is 792.