Main concepts from today:

- Linear equation and System of linear equations (Linear System)
- Solution and Solution Set
- Consistent/Inconsistent system
- Number of solutions in a linear system
- Augmented matrix vs Coefficient matrix

Here is the example at the end of today. You should go over the calculation carefully and consider the questions I pose at the end.

Solve the following linear system

\[
\begin{align*}
x_1 - 2x_2 + 3x_3 &= 9 \\
-x_1 + 3x_2 &= -4 \\
2x_1 - 5x_2 + 5x_3 &= 17
\end{align*}
\]

\[\begin{bmatrix}
1 & -2 & 3 & 9 \\
0 & 1 & 3 & 5 \\
0 & -1 & -1 & -1
\end{bmatrix}\]

\[
\begin{align*}
x_1 - 2x_2 + 3x_3 &= 9 \\
x_2 + 3x_3 &= 5 \\
-x_2 - x_3 &= -1
\end{align*}
\]

\[\begin{bmatrix}
1 & -2 & 3 & 9 \\
0 & 1 & 3 & 5 \\
0 & -1 & -1 & -1
\end{bmatrix}\]

\[
\begin{align*}
x_1 - 2x_2 + 3x_3 &= 9 \\
x_2 + 3x_3 &= 5 \\
2x_3 &= 4
\end{align*}
\]

\[\begin{bmatrix}
1 & -2 & 3 & 9 \\
0 & 1 & 3 & 5 \\
0 & -0 & 2 & 4
\end{bmatrix}\]

Remark: At the end of step 2, we obtain a new system that is easier to solve than then original one.

3. We should now be able to solve for the variables, as follows.

From \(2x_3 = 4\), it is easy to see that \(x_3 = 2\).
Substitute $x_3 = 2$ into $x_2 + 3x_3 = 5$ to obtain $x_2 = -1$
Substitute $x_3 = 2$ and $x_2 = -1$ into $x_1 - 2x_2 + 3x_3 = 9$ to obtain $x_1 = 1$.

4. Thus, the solution is $x_1 = 1, x_2 = -1, x_3 = 2$ or $(x_1, x_2, x_3) = (1, -1, 2)$.

Now consider the following:

1. Can you observe any special feature of the augmented matrix associated with the final system?

2. $(x_1, x_2, x_3) = (1, -1, 2)$ is in fact the solution to the final system, which is

\[
\begin{align*}
    x_1 & -2x_2 & +3x_3 &= 9 \\
    x_2 & +3x_3 &= 5 \\
    2x_3 &= 4
\end{align*}
\]

how can we guarantee that the same triple is also a solution to the original system? Furthermore, how can we guarantee that the two systems have the same solution set?

3. How do we know that $(x_1, x_2, x_3) = (1, -1, 2)$ is the only solution?