$A = [a_1, a_2, ..., a_n]$ is an $m \times n$ matrix

**Existence**

- $x \mapsto Ax$ is onto
  - $Ax = b$ is consistent for all $b$ in $\mathbb{R}^m$
  - Every $b$ in $\mathbb{R}^m$ is a linear combination of the columns of $A$
  - $\text{Span}\{a_1, ..., a_n\} = \mathbb{R}^m$
    - i.e. the columns of $A$ span $\mathbb{R}^m$
  - $A$ has a pivot in every ROW

**Uniqueness**

- $x \mapsto Ax$ is one to one
  - $Ax = 0$ has only trivial solution
  - The columns of $A$ are linearly independent
  - $A$ has a pivot in every COLUMN