Question 1

Suppose $g : \mathbb{R}^2 \to \mathbb{R}^2$ such that $Dg(x) = \begin{bmatrix} 0 & -2 \\ 1 & 0 \end{bmatrix}$ and $f : \mathbb{R}^2 \to \mathbb{R}^2$ such that $Df(g(x)) = \begin{bmatrix} 0 & 1 \\ 1 & -3 \end{bmatrix}$. Then $D(f \circ g)(x)$ is

A. $\begin{bmatrix} 1 & 0 \\ -3 & -2 \end{bmatrix}$

B. $\begin{bmatrix} -2 & 6 \\ 0 & 1 \end{bmatrix}$

C. $\begin{bmatrix} 0 & -1 \\ 2 & -4 \end{bmatrix}$

D. $D(f \circ g)(x)$ is undefined
Directional Derivative

Definition

If $f : \mathbb{R}^n \rightarrow \mathbb{R}$ is differentiable then the **directional derivative** of $f$ at $x_0$ along the unit vector $u = (u_1, \ldots, u_n)$ is given by

$$(D_uf)(x_0) = \nabla f(x_0) \cdot u = \left[ \frac{\partial f}{\partial x_1}(x_0) \right] u_1 + \left[ \frac{\partial f}{\partial x_2}(x_0) \right] u_2 + \cdots + \left[ \frac{\partial f}{\partial x_n}(x_0) \right] u_n$$
Find the rate of change of \( f(x, y, z) = 2xy - e^{x^2y} + z^2 \) at \((2, 1, 0)\) in the direction of \( \mathbf{u} = (1, -1, 1) \).
Here, we have \( \nabla f(x, y, z) = (2y - 2xye^{x^2y}, 2x - x^2e^{xy}, 2z) \) so
\[
\nabla f(2, 1, 0) = (2 - 4e^4, 4 - 4e^4, 0).
\]

A. \(-2\)
B. \(-2\) \(\sqrt{3}\)
C. \(6 - 8e^4\)
D. \(6 - 8e^4\) \(\sqrt{3}\)
Question 3

Given a differentiable function $f : \mathbb{R}^n \to \mathbb{R}$ such that $\nabla f(x_0) \neq 0$. Let $u$ be a unit vector. Find the direction for which $(D_u f)(x_0)$ increases most rapidly/fastest.

A. Along the same direction of $\nabla f(x_0)$
B. Along the opposite direction of $\nabla f(x_0)$
C. Along the direction that is perpendicular to $\nabla f(x_0)$

The fastest rate of increasing is given by $\|\nabla f(x_0)\|$
Given a differentiable function \( f : \mathbb{R}^n \to \mathbb{R} \) such that \( \nabla f(x_0) \neq 0 \). Let \( u \) be a unit vector. Find the direction for which \((D_uf)(x_0)\) decreases most rapidly/fastest.

A. Along the same direction of \( \nabla f(x_0) \)
B. Along the opposite direction of \( \nabla f(x_0) \)
C. Along the direction that is perpendicular to \( \nabla f(x_0) \)

The fastest rate of decreasing is given by \(-\|\nabla f(x_0)\|\)
Given a differentiable function $f : \mathbb{R}^n \to \mathbb{R}$ such that $\nabla f(x_0) \neq 0$. Let $u$ be a unit vector. Find the direction for which $f$ remains unchanged.

A. Along the same direction of $\nabla f(x_0)$
B. Along the opposite direction of $\nabla f(x_0)$
C. Along the direction that is perpendicular to $\nabla f(x_0)$
Example

Jon Snow is standing at the point $(0, 1)$ in the Haunted Forest beyond the Wall. In the North, the temperature at the point $(x, y)$ is given by

$$T(x, y) = 30e^{-(x-1)^2-(2y-1)^2}.$$ 

Jon is freezing, which direction should he move to warm himself up the fastest?