Problem 1. For each part, you are given a function $f$, a point $x_0$, and a vector $u$. Compute $(D_u f)(x_0)$, the directional derivative of $f$ at $x_0$ in the direction $u$. Not all $u$ is a unit vector.

a. $f(x, y) = x + 2xy - 3y^2$; $x_0 = (1, 2); u = (3, 4)$
b. $f(x, y) = \ln(\sqrt{x^2 + y^2})$; $x_0 = (1, 0); u = (2, 1)$
c. $f(x, y) = xy^2 + y^2z + z^3x$; $x_0 = (4, -2, -1); u = (1, 3, 2)$
d. $f(x, y, z) = x^{yz}$; $x_0 = (e, e, 0); u = (12, 3, 4)$
e. $f(x, y, z) = e^{-x^2 - y^2-z^2}$; $x_0 = (1, 10, 100); u = (1, -1, -1)$

Problem 2. For each part, you are given a surface $S$ and a point $P$. Find the equation for the tangent plane to $S$ at the point $P$.

a. $S : x^2 + y^2 + 3z^2 = 107$; $P = (1, \sqrt{3}, 1)$
b. $S : z = x^2 + 3y^3 + \sin(xy)$; $P = (1, 0, 1)$
c. $S : x^2y + xy^2 + yz^2 = 3$; $P = (1, 1, 1)$
d. $S : x^2 + y^2 - z^2 = 18$; $P = (3, 5, -4)$

Problem 3. Let

$$f(x, y) = (x^2 + y^2)e^{-(x^2+y^2+10)}.$$ 

Find the rate of change of $f$ at $(2, 1)$ in the direction pointing toward the origin.

Problem 4. The temperature in space at position $(x, y, z)$ is given by

$$T(x, y, z) = x^2 + y^2 - 3z^2.$$ 

Suppose an eagle is at position $(0, 1, 1)$.

a. What is the direction(s) that it must follow to increase the temperature as fast as possible?
b. What is the direction(s) that it must follow to maintain the same temperature?
Problem 5. Let $f(x, y) = e^{xy} \sin(x + y)$.

a. In what direction(s), starting at $(0, \pi/2)$, that $f$ is decreasing the fastest?

b. In what direction(s), starting at $(0, \pi/2)$, that $f$ is changing at 50% of its maximum rate?

Problem 6. Consider the function

$$f(x, y) = \frac{x^2 - y^2}{x^2 + y^2}.$$ 

Let $(x_0, y_0)$ be an arbitrary point in the first quadrant.

a. Find the direction(s) in which the directional derivative of $f$ at $(x_0, y_0)$ is zero.

b. Describe the level curves of $f$ in terms of the results from previous part.