1) Given a word \( w = w_1 \ldots w_n \in \{1, \ldots, k\}^* \), let \( |w| = n \) denote the length of \( w \), \( Z(w) = \prod_{i=1}^{n} z_{w_i} \), and \( \text{lev}(w) = \{|i : w_i = w_{i+1}\}| \). Define a ring homomorphism \( \phi \) on \( \Lambda(x_1, x_2, \ldots) \) by defining
\[
\phi(e_n) = (-1)^{n-1} p_n(x_1, \ldots, x_k)(x-1)^{n-1}
\]
where \( p_k \) is the power symmetric function. Show that
\[
\phi(\sum_{n \geq 0} h_n t^n) = \sum_{w \in \{1, \ldots, k\}^*} x^{\text{lev}(w)} Z(w) t^{|w|}.
\]

(2) Do problem 3.9 in the book.

(3) Do problem 3.10 in the book.

(4) Do problem 3.11 in the book.


(6) Let \( E_{n}^{(3)} \) denote the set of permutations \( \sigma = \sigma_1 \ldots \sigma_n \in S_n \) such that \( \sigma_i > \sigma_{i+1} \) if and only if \( i \equiv 0 \mod 3 \). Find the generating function
\[
1 + \sum_{n \geq 1} \frac{t^n}{n!} |E_{n}^{(3)}|.
\]
(Hint: Modify the proof of the generating function for up-down permutations by finding separate expressions for the generating functions
\[
\sum_{n \geq 0} \frac{t^{3n}}{(3n)!} |E_{3n}^{(3)}|, \quad \sum_{n \geq 0} \frac{t^{3n+1}}{(3n+1)!} |E_{3n+1}^{(3)}|, \quad \text{and} \quad \sum_{n \geq 0} \frac{t^{3n+2}}{(3n+2)!} |E_{3n+2}^{(3)}|.
\]