(1) Prove Theorems 5.7, 5.8, and 5.10 in the book.

(2) Do problem 4.5.

(3) Do problem 4.7.

(4) Do Problem 5.3.

(5) Let $f(n, k)$ be the number of ways of arranging $n$ children in a circles such that each circle consists of at least 2 children holding hands and there is child in the center of each circle. Find the generating function

$$1 + \sum_{n \geq 3} \frac{t^n}{n!} \sum_{k \geq 1} f(n, k)x^k.$$