Math 10B Test 1 Solutions **100 pts** **October 21, 2009** **Professor Evans**

Directions: Answers alone are not sufficient. Show all work. Each problem is 20 points.

(1) Let $f(x)$ denote the function e^{x^2}. Recall that $e = 2.71828\ldots$. Jane partitioned the interval $[0,1]$ into 175 equal parts and then computed a corresponding left Riemann sum for the function $f(x)$ above. Carefully explain why the value of Jane's Riemann sum must be within .01 of the exact value of $\int_0^1 f(x)\, dx$.

Solution (1) Since f is increasing, the exact area $\int_0^1 f(x)\, dx$ lies between the left Riemann sum LRS (underestimate) and the right Riemann sum RRS (overestimate). Thus it suffices to show that RRS minus LRS is less than .01. This is true, because the RRS minus LRS equals $(f(1) - f(0))(b - a)/n = (e^1 - e^0)(1/175) = (e - 1)/175$, which is less than .01 (as can be seen by cross multiplying).

(2) Find the exact area of the (finite) region bounded between the parabola $y = x^2$ and the line $y = 3x$. *Hint*: The answer is between 2 and 10.

Solution (2) The region is bounded between the intersection points $x = 0$ and $x = 3$. In the interval $[0, 3]$, the line is above the parabola, so the area is $\int_0^3 (3x - x^2)\, dx = 27/2 - 27/3 = 4.5$.

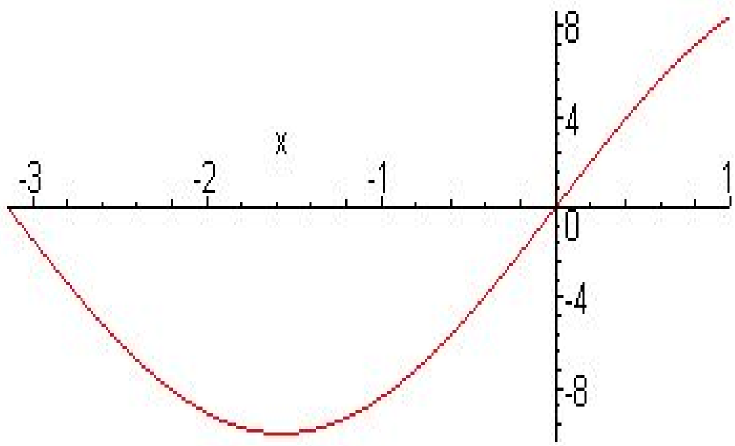
(3) From point A on the runway, a taxiing jet accelerates at a constant 10 feet per second^2. The jet lifts off at point B. Given that the jet's speed is 100 feet per second at point A and 200 feet per second at point B, find the distance between points A and B.

Solution (3) By taking the antiderivative of the constant acceleration 10, we see that the speed $v(t)$ equals $10t + C$. Plug in the time $t = 0$ (at point A) to get $C = 100$, so $v(t) = 10t + 100$. Thus $s(t) = 5t^2 + 100t$. Set $v(t) = 200$ and solve for t in order to see that the time t at point B is $t = 10$. The desired distance between A and B is thus $s(10) = 1500$.

(4) Find the exact value of the integral $\int_0^1 (x+1)\sqrt{x^2 + 2x}\, dx$. *Hint*: First find the antiderivative of $(x + 1)\sqrt{x^2 + 2x}$ and then slash it from $x = 0$ to $x = 1$. The answer is between 0 and 4.

Solution (4) Find an antiderivative by the substitution method. Choose $w = x^2 + 2x$, so $dw = 2(x + 1)dx$. An antiderivative is thus $\int \sqrt{w}(1/2)dw = w^{3/2}/3 = (x^2 + 2x)^{3/2}/3$. Slashing from $x = 0$ to $x = 1$, we get the answer $3^{3/2}/3 = \sqrt{3}$.

(5) For a certain function $F(x)$, the graph of its derivative $F'(x)$ is pictured below. Given that $F(0) = 100$, estimate the value of $F(-1)$. Explain how you got your estimate.



Solution (5) The integral from -1 to 0 of $F'(x)$ is approximately -4.5 (estimated from the picture), so by the Fundamental Theorem of Calculus, $F(0) - F(-1) = -4.5$ (rough estimate). Thus $F(-1) = F(0) + 4.5 = 104.5$.