**Math 10B Test 2** **100 pts** **November 23, 2009** **Professor Evans**

**Directions:** Answers alone are not sufficient. Show all work. Each problem is worth 20 points.

**(1)** On January 2, 2008, Jack was given a \$100 savings account and Jane was given a \$60 savings account. On January 2, 2009, Jane was given another \$60 savings account. The accounts pay an annual interest rate of 50%. *True* or *False*:
If Jack and Jane withdraw their money on January 2, 2010, they will each have the same amount. *Justify.*

Solution: Jack ends up with $100(1.50)^2 = 225$ dollars. Jane ends up with $60(1.50)^2 + 60(1.50) = 225$ dollars. Hence the answer is True.

**(2)** The base of a loaf of bread is the region inside the circle $x^2 + y^2 = 1$. Each cross-section perpendicular to the $x$-axis has the shape of a square. (Think of paper-thin square-shaped slices of bread.) Find the volume of the loaf of bread. *Hint*: The answer is between 1 and 8.

Solution: The cross section at any given $x$ is a square of side length $2\sqrt{1-x^2}$. The area of this square is $4(1-x^2)$. Hence the volume of the loaf is $\int_{-1}^{1} 4(1-x^2)dx = 16/3$.

**(3)** Two cones E and F have the same shape, and their bases are octagons. Cone E has a height of 8 inches and Cone F has a height of 5 inches. *True* or *False*:
**(A)** The volume of cone E is more than 4 times the volume of cone F.
**(B)** The area of cone E's octagonal base is more than 3 times the area of cone F's octagonal base.
**(C)** The perimeter of cone E's octagonal base is more than twice the perimeter of cone F's octagonal base.
*Justify your three answers very briefly.*

Solution: The cones have the same shape, so all three dimensions are in the ratio of (8 in / 5 in) for big/small.
**(A)** $(8/5)^3 = 512/125 > 4$, so statement (A) is true. (Recall that volume units are cubic inches.)
**(B)** $(8/5)^2 = 64/25 < 3$, so statement (B) is false. (Recall that area units are square inches.)
**(C)** $(8/5) < 2$, so statement (C) is false. (Recall that perimeter units are inches.)

**(4)** Define the function $f(x) = 1/(x^2 + x)$ for $x > 0$. Using the **method of partial fractions**, show in detail that the antiderivative of $f(x)$ is equal to $\ln(1 - 1/(x+1)) + C$.
*Hint*: Recall the rule of logs: $\ln(S) - \ln(T) = \ln(S/T)$

Solution: $f(x) = A/x + B/(x+1)$, where $1 = A(x+1) + Bx$. Thus $A = 1$ and $B = -1$, so $f(x) = 1/x - 1/(x+1)$. The antiderivative of $f(x)$ is $\ln(x) - \ln(x+1) = \ln(x/(x+1)) = \ln(1 - 1/(x+1))$. To see that $1 - 1/(x+1) = x/(x+1)$, get a common denominator on the left side.

**(5)** Define the function $f(x) = 1/(x^2 + x)$ for $x > 0$, as in Problem 4.
*Multiple choice*:

**(A)** The area of the region under $f(x)$ between $x = 0$ and $x = 1$ is equal to
(a) infinity (b) 1 (c) ln(2) (d) 2 (e) none of these
**(B)** The area of the region under $f(x)$ to the right of $x = 1$ is equal to
(a) infinity (b) 1 (c) ln(2) (d) 2 (e) none of these
*Justify your two answers.*

Solution: **(A)** The area is the limit of $G(a) = \ln(1 - 1/(x+1))|_a^1$ as $a \to 0$, where $a > 0$. Note that $G(a) = \ln(1/2) - \ln(1 - 1/(a+1))$. Since $G(a)$ gets arbitrarily large as $a \to 0$, the limit is infinity as $a \to 0$, so answer for the area is "(a) infinity".
**(B)** The area is the limit of $H(b) = \ln(1 - 1/(x+1))|_1^b$ as $b \to \infty$, where $b > 1$.
Note that $H(b) = \ln(1 - 1/(b+1)) - \ln(1/2)$. Since $H(b)$ approaches $0 - \ln(1/2) = \ln(2)$ as $b \to \infty$, the answer for the area is "(c) ln(2)".