Math 10B Final Exam Solutions 150 pts December 8, 2009 Professor Evans

Directions: Answers alone are not sufficient. Justify, and show all work. The multiple part problems 4,6,7 are worth 18 points each. The other six problems are worth 16 points each.

(1) A man was murdered in a 34.6 degree Fahrenheit meat locker. At noon, the corpse temperature was 38.6, and an hour later, it had dropped to 36.6. The man was murdered at
(a) 6 am (b) 7 am (c) 8 am (d) 9 am (e) none of these times
Solution: Let $T(t)$ denote the temperature of the corpse at time t, with noon being chosen as the "zero hour" $t = 0$. The solution to the differential equation $dT/dt = k(34.6 - T)$ is $T(t) = 34.6 + Be^{-kt}$. Plug in $t = 0$ to get $38.6 = 34.6 + B$, so $B = 4$ and $T = 34.6 + 4e^{-kt}$. Plug in $t = 1$ to get $36.6 = 34.6 + 4e^{-k}$. Thus $e^{-k} = 1/2$ and $T = 34.6 + 4(1/2)^t$. At the time of the murder, $98.6 = 34.6 + 4(1/2)^t$, so $16 = (1/2)^t$. Thus $t = -4$, so the murder took place at 8 am.

(2) An upright vase of height 16 is formed by rotating the curve $y = x^4 \quad (0 \le x \le 2)$ around the y-axis. Using an integral, find the volume of water that it takes to fill the vase.
Solution: Volume (via horizontal slicing) $= \int_0^{16} \pi(y^{1/4})^2 dy = 128\pi/3$.

(3) A train is going 80 ft per sec on straight track. At Irvine station, it suddenly brakes, with a constant deceleration of 10 ft per sec^2. How far from Irvine station will the train travel before coming to a stop?
Solution: We are given that $a = -10$, and integrating yields $v(t) = -10t + C$. Plug in $t = 0$ to obtain $C = 80$, so $v(t) = 80 - 10t$. Integrate again to get the position $s(t) = 80t - 5t^2$, given that Irvine station is the zero position. Since $v = 0$ when $t = 8$, the stopping distance is $s(8) = 320$ ft.

(4) A solid wood cone E has height 8 feet, and its pentagonal base has area 24 square feet. Cone E is sliced parallel to its base to form a smaller cone F whose height is 6 feet. Answer the three questions below, and justify VERY briefly.
(A) What is the area of the pentagonal base of cone F?
(B) When the surface of cone E was painted last year, it took 16 cans of paint. How many cans would it take to repaint the surface of the smaller cone F?
(C) What is the volume of cone F?
Solution: ALL dimensions for cone F are reduced by a factor of 3/4. Thus the base area of cone F is $24 * (3/4)^2 = 27/2$ sq ft. The surface area is also reduced by the same factor, so the required number of cans of paint is $16 * (3/4)^2 = 9$ cans. The volume of cone F is one third of the base area times the height, namely $(1/3)(27/2)(6) = 27$ cubic ft.

(5) Define the function $f(x) = 1/(x^2 + 2x)$ for $x > 0$. Using the method of partial fractions, show in detail that the antiderivative of $f(x)$ is equal to $.5\ln(1 - \frac{2}{x+2}) + C$.
Solution: $f(x) = .5(\frac{1}{x} - \frac{1}{x+2})$, by the partial fraction decomposition. Thus $\int f(x)dx = .5(\ln x - \ln(x+2)) + C = .5\ln(\frac{x}{x+2}) + C = .5\ln(1 - \frac{2}{x+2}) + C$, where the last formula follows by getting a common denominator.

(6) Define the function $f(x) = 1/(x^2 + 2x)$ for $x > 0$, as in Problem 5.
(A) The area of the region under $f(x)$ between $x = 0$ and $x = 2$ is equal to
(a) infinity (b) ln 2 (c) (ln 2)/2 (d) ln .5 (e) none of these

(B) The area of the region under $f(x)$ to the right of $x = 2$ is equal to
(a) infinity (b) ln 2 (c) (ln 2)/2 (d) ln .5 (e) none of these
Solution:(A) infinity (B) (ln 2)/2 (Cf. Midterm 2 solutions)

(7) Let γ denote the area of the region under the curve $y = x^2$ between $x = 3$ and $x = 9$.
(A) Estimate the value of γ using the MIDPOINT rule with $n = 3$ subintervals.
(B) Find the EXACT value of γ.
(C) Explain why you could predict in advance (without estimating or computing γ) that the answer to part (A) would be less than the answer to part (B).
Solution:(A) $2(4^2 + 6^2 + 8^2) = 232$ (B) The integral is $9^3/3 - 3^3/3 = 234$ (C) The three trapezoids of width 2 whose tops are tangent to the curve at $x = 4, 6, 8$, respectively, all lie BELOW the curve $y = x^2$.

(8) Find the solution $P(t)$ to the differential equation $dP/dt = 2tP^2$, given that $P(0) = 1/2$. (You may assume that t lies in the interval $(-1, 1)$.)
Solution: $dP/P^2 = 2tdt$, so integration gives $-1/P = t^2 + C$, and inverting then gives $P = -1/(t^2 + C)$. Plug in $t = 0$ to get $1/2 = -1/C$. Thus $C = -2$, so $P(t) = 1/(2 - t^2)$.

(9) Find the area under the curve $y = x\sin(2x)$ between $x = \pi$ and $x = 3\pi/2$.
Hint: $\cos(3\pi) = -1$.
Solution: This curve lies above the x-axis. Integration by parts yields $(1/4)\sin(2x) - (1/2)x\cos(2x)$. Slashing from π to $3\pi/2$ yields the area $5\pi/4$.