Announcements

- Definition list for Midterm 2 has been posted on the website.
Last time we saw the Prisoner’s Dilemma.

The payoff matrix was given by:

$$
\begin{pmatrix}
C & D \\
C & (3, 3) & (0, 4) \\
D & (4, 0) & (1, 1)
\end{pmatrix}
$$
Pure Strategic Equilibria

- **Def.** A pair of pure strategies $x, y$ is a pure strategic equilibrium if

$$G_1(x, y) \geq G_1(x', y)$$

for any other pure strategy $x'$ for Player I and

$$G_2(x, y) \geq G_2(x, y')$$

for any other pure strategy $y'$ for Player II. $G$ represents the payoff function of some bimatrix game.

- This is saying that $x$ is the greatest entry in row $y$ of Player I’s payoff matrix and $y$ is the greatest entry in row $x$ of Player II’s payoff matrix.
Pure Strategic Equilibria

- The Prisoner’s Dilemma has one pure strategic equilibrium, corresponding to both players defecting.
- One of the desirable features of a strategic equilibrium is that it is **self-enforcing**: even without a binding agreement between players, it is in both player’s interests to stay at a strategic equilibrium.
Pure strategic equilibria do not need to be unique:

\[
\begin{pmatrix}
(2, 3) & (0, -3) \\
(-2, 0) & (10, 10)
\end{pmatrix}
\]

And they do not necessarily exist:

\[
\begin{pmatrix}
(3, -3) & (0, 0) \\
(1, -1) & (4, -4)
\end{pmatrix}
\]
**Def.** A pair of (mixed) strategies $p, q$ is a strategic equilibrium if

$$G_1(p, q) \geq G_1(p', q)$$

for any other strategy $p'$ for Player I and

$$G_2(p, q) \geq G_2(p, q')$$

for any other strategy $q'$ for Player II. $G$ represents the payoff function of some bimatrix game.
Let $G$ be a bimatrix game with payoff matrices $A$ and $B$ for Players I and II respectively.

**Def.** A row $x'$ of $A$ is dominated by row $x$ if $A(x, y) \geq A(x', y)$ for all $y$.

**Def.** A column $y'$ of $B$ is dominated by column $y$ if $B(x, y) \geq B(x, y')$ for all $x$.

For example:

\[
\begin{pmatrix}
(3, 5) & (2, 4) \\
(4, 4) & (1, 3)
\end{pmatrix}
\]

Neither row dominates the other, but the first column dominates the second.

**Remark.** Unlike for zero-sum games, to dominate a column we want to find another column that is bigger than or equal to it.
What are the strategic equilibria for this game?
Well, note that in the right-most position that by domination Player II will always select “down” instead of “right”.

Player I knows this. So Player I will select “down” not “right” in the second last position.

... 

Continuing this, we eventually see that Player I chooses “down” on the first move.
This is not a very satisfying answer. It does not match what we expect real players to do.
If we were serious about this, the first step is to determine scientifically what real players actually do.


In a 6-turn variant of the Centipede game, they found that only 1% of games ended on turn 1.

*What is the value of our theory if it has no predictive power?*
We should not be overly critical of the theory: we just need to work a bit harder.

In reality, we know there is a chance our opponent will cooperate. So we are likely to cooperate a little ourselves.

McKelvey and Palfrey realised this (and observed it in their experiments), so devised the following model: some proportion of people are altruists and will cooperate.

When we start a game, we do not know if our opponent is an altruist. Let us put this information into our game.
Centipede Game: More Realistic Model

This game is harder to study, but it does have a strategic equilibrium where the players cooperate for many turns.
Find the strategic equilibria, both pure and mixed, for the bimatrix game below.

\[
\begin{pmatrix}
(3, 3) & (0, 2) \\
(2, 1) & (5, 5)
\end{pmatrix}
\]
It is easy to find the two pure strategic equilibria: the top left and bottom right entries.

But what about mixed strategies equilibria?

One useful approach is to look for an equalizing strategic equilibrium: try to find an equalizing strategy for each player on their opponent’s matrix.

Why does this work? Well, under such a strategy, your opponent’s payoff is the same regardless of their move. So they have no incentive to switch (which is exactly a strategic equilibrium).
For the example above, we can find an equalizing strategic equilibrium: \( p = \left( \frac{4}{5}, \frac{1}{5} \right) \) and \( q = \left( \frac{5}{6}, \frac{1}{6} \right) \). The corresponding payoffs are \( \left( \frac{5}{2}, \frac{13}{5} \right) \).

How do these three equilibria compare to the safety levels for this game?

Can the payoffs in a strategic equilibrium ever be less than the safety levels? No. (Why?)