MA152 Spring 2017

Homework 7

Due: 7th June at 4PM in APM basement

1. Draw the TU and NTU-feasible sets for the following bimatrix game.
   Indicate the Pareto optimal curve in both diagrams.
   \[
   \begin{pmatrix}
   (0, 4) & (3, 2) \\
   (4, 0) & (2, 3)
   \end{pmatrix}
   \]

2. Find the TU solution and sidepayment for the bimatrix game
   \[
   \begin{pmatrix}
   (3, 1) & (4, 3) & (−5, −5) \\
   (0, 5) & (1, 0) & (5, 0)
   \end{pmatrix}
   \]

3. For each of the following bimatrix games, find the NTU solution given
   that the threat point is (0, 0) (use the Nash approach, and not \(\lambda\)-
   transfer).
   (a)
   \[
   \begin{pmatrix}
   (1, 5) & (0, 0) \\
   (1, 1) & (3, 0)
   \end{pmatrix}
   \]
   (b)
   \[
   \begin{pmatrix}
   (1, 5) & (0, 0) \\
   (0, 0) & (2, 4)
   \end{pmatrix}
   \]

4. Let \(S = \{(x, y) : 0 \leq y \leq 4 - x^2\}\) be an NTU-feasible set.
   (a) Find the NTU solution if the threat point is \((u^*, v^*) = (0, 0)\).
   (b) Find the NTU solution if the threat point is \((u^*, v^*) = (0, 1)\).
5. (Not to be handed in.) Consider a three-player game with the following characteristic function: 
\[ v(\{1\}) = 1, \quad v(\{2\}) = 0, \quad v(\{3\}) = 2, \quad v(\{1, 2\}) = 2, \quad v(\{2, 3\}) = 3, \quad v(\{1, 3\}) = 4, \quad v(\{1, 2, 3\}) = 7. \] 
Compute the Shapeley values for each player.

6. (Not to be handed in.) Consider the following 3-person game of perfect information. Let \( S = \{1, 2, \cdots, 10\} \). First Player 1 chooses \( i \in S \). Then Player 2, knowing \( i \), chooses \( j \in S, \ j \neq i \). Finally Player 3, knowing \( i \) and \( j \), chooses \( k \in S, \ k \neq i, \ k \neq j \). The payoff given these three choices is \( (|i - j|, |j - k|, |k - i|) \). Find the coalitional form of the game.

7. (Not to be handed in.) Consider the three player game where each player simultaneously announces 0 or 1. Let \( x \) be the sum of the three announced numbers. If \( x \) is a multiple of 3, the payoff is \( x \) to Player 1 and 0 to the other players. If \( x \) is 1 more than a multiple of 3, the payoff is \( x \) to Player 2 and 0 to the other players. Finally if \( x \) is 2 more than a multiple of 3, the payoff is \( x \) to Player 3 and 0 to the other players. Find the coalitional form of the game.