1. (a) State the formal definition of N and P-positions (also called the characteristic property).

(b) For the subtraction game, where players may remove 2, 3 or 5 chips on their turn, determine which positions are N-positions and which are P-positions.

2. (a) For a progressively bounded, directed graph \( G \), state the definition of the Sprague–Grundy function \( g \).

(b) For the graph game below, compute the Sprague–Grundy value for every position.

3. Consider a game where there are piles of chips, and on a player’s turn they may either: remove any positive number of chips from a single pile, or divide any single pile into exactly two new piles. Compute the Sprague–Grundy value for this game for positions that consist of a single pile, with between 0 and 12 chips.

4. The game of Kayles is played with a set of coins arranged in a line, where each coin has two sides: ‘H’ and ‘T’. On a player’s turn, they may either flip one ‘H’ into a ‘T’, or they may take two adjacent ‘H’-s and flip both into ‘T’-s. For the position below, determine all winning moves (ie. moves to P-positions):

“HHHHHHHHHHHTHHHH”

(ie. 11 ‘H’s, 1 ‘T’ then 4 ‘H’-s)

The last player to make a move is the winner (the normal play rule).

(Hint: Translate the game into a game with piles of chips.)