Instructions
1. Write your Name and PID on the front of your Blue Book.
2. No calculators or other electronic devices are allowed during this exam.
3. You may use the textbook during this exam.
4. Read each question carefully, and answer each question completely.
5. Write your solutions clearly in your Blue Book.
   (a) Carefully indicate the number and letter of each question and question part.
   (b) Present your answers in the same order as they appear in the exam.
   (c) Start each numbered problem on a new side of a page.
6. Show all of your work and justify all your claims. No credit will be given for unsupported answers, even if correct.

0. Carefully read and complete the instructions at the top of this exam sheet and any additional instructions written on the chalkboard during the exam.

1. (10 points) Consider the set of $n \times n$ real matrices with determinant 1 or $-1$. Is this set a group with respect to (matrix) multiplication? Justify your answer.
2. (10 points) Is the additive group $\{a + b\sqrt{3} : a, b \in \mathbb{Z}\}$ cyclic? Justify your answer.
3. (10 points) Consider the homomorphism $\varphi : \mathbb{Z}_8 \rightarrow \mathbb{Z}_{16}$ such that $\varphi(1) = 4$. Find $|\ker(\varphi)|$. Justify your answer.
4. (10 points) Let $\varphi : \mathbb{Z}_{18} \rightarrow \mathbb{Z}_3$ be the homomorphism such that $\varphi(1) = 2$. Find the kernel $K$ of $\varphi$ and list the cosets of $K$. Justify your answer.
5. (10 points) Find the order of the factor group $\mathbb{Z}_8 \times \mathbb{Z}_6/\langle(2, 3)\rangle$. Justify your answer.

(This exam is worth 50 points.)