Instructions

1. Write your Name and PID on the front of your Blue Book.
2. No calculators or other electronic devices are allowed during this exam.
3. You may use the textbook during this exam.
4. Read each question carefully, and answer each question completely.
5. Write your solutions clearly in your Blue Book.
   (a) Carefully indicate the number and letter of each question and question part.
   (b) Present your answers in the same order as they appear in the exam.
   (c) Start each numbered problem on a new side of a page.
6. Show all of your work and justify all your claims. No credit will be given for unsupported answers, even if correct.

0. Carefully read and complete the instructions at the top of this exam sheet and any additional instructions written on the chalkboard during the exam.

1. (10 points) Find the multiplicative inverse of 9 in $\mathbb{Z}_{11}$. Justify your answer.
2. (10 points) How many homomorphisms are there of $\mathbb{Z}_6 \oplus \mathbb{Z}_6$ into $\mathbb{Z}_4$. Justify your answer.
3. (10 points) What is the order of the group of units of $\mathbb{Z}_6 \oplus \mathbb{Z}_3 \oplus \mathbb{Z}_4$? Justify your answer.
4. (10 points) Find all the units of the ring $\mathbb{Z}_{21}$. Justify your answer.
5. (10 points) Is $R = \{a + bi : a, b \in \mathbb{Z}_3, i^2 = -1\}$ an integral domain? Justify your answer.

(This exam is worth 50 points.)