Midterm 2vS
Math 18, Section A
Name (Print):
May 19, 2017
Time Limit: 50 Minutes
PID:

- DO NOT begin working, or even open this packet, until instructed to do so.
- You should be in your assigned seat, unless instructed otherwise by Ed or one of the TAs.
- Enter all requested information on the top of this page, and put your name on the top of every page, in case the pages become separated.
- You may use a two-sided page of notes on this exam.
- You may not use your books, additional notes, or any calculator on this exam.

You are required to show your work on each problem on this exam. The following rules apply:

- Organize your work, in a reasonably neat and coherent way, in the space provided. Work scattered all over the page without a clear ordering will receive very little credit.
- Mysterious or unsupported answers will not receive full credit. Unless otherwise directed in the statement of the problem, a correct answer, unsupported by calculations, explanation, or algebraic work will receive little or no credit. An incorrect answer supported by substantially correct calculations and explanations might still receive partial credit.

| Question | Points |
| :---: | :---: |
| 1 | 10 |
| 2 | 10 |
| 3 | 10 |
| 4 | 10 |
| Total: | 40 |

## DO NOT turn this page until instructed to do so

1. Let $A=\left[\begin{array}{llll}1 & 0 & 3 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0\end{array}\right]$ and let $a, b, c, d$ be some numbers.
(a) (7 points) Find $A^{-1}$.
(b) (3 points) Solve the equation $A \vec{x}=(a, b, c, d)$. You may answer in terms of $a, b, c$ and $d$.
2. (10 points) Let $\mathbb{P}_{2}$ be the vector space of polynomials with degree at most 2 , and let $\mathcal{B}=$ $\left\{1+x, 1+x^{2}, x+x^{2}\right\} . \mathcal{B}$ is a basis for $\mathbb{P}_{2}$ (and you don't need to prove this). Let $T: \mathbb{R}^{3} \rightarrow \mathbb{R}^{3}$ satisfy $T\left(\left[a_{0}+a_{1} x+a_{2} x^{2}\right]_{\mathcal{B}}\right)=\left(a_{0}, a_{1}, a_{2}\right)$. Find the matrix corresponding to $T$.
3. (10 points) Let $A$ be a matrix which is row-equivalent to $\left[\begin{array}{cccc}1 & 2 & 0 & 3 \\ 0 & 0 & 5 & 6 \\ 0 & 0 & 0 & 0\end{array}\right]$, and let $T: \mathbb{R}^{3} \rightarrow \mathbb{R}^{7}$ be a linear transformation such that $T(\vec{x})=\overrightarrow{0}$ if and only if $\vec{x}=A \vec{y}$ for some $\vec{y} \in \mathbb{R}^{4}$. Find the dimension of the range of $T$.
4. (a) (5 points) Let $P$ be a parallelopiped in $\mathbb{R}^{3}$. Suppose that one of its vertices is $(0,1,0)$, and the three vertices adjacent to that one are $(0,1,2),(1,1,1)$ and $(1,0,-1)$. Find the volume of $P$.
(b) (5 points) Let $A$ and $B$ be invertible matrices with $\operatorname{det} A=4$ and $\operatorname{det} B=2$. Find $\operatorname{det}\left(A^{2} B^{-1}\right)$.
