Math 20F **March 17, 2015** | **Final Examination** **Version A** | **Name:** ____________ **PID:** ____________

Instructions

1. **Write your *Name*, *PID*, *Section*, and *Exam Version* on the front of your Blue Book.**
2. **No calculators or other electronic devices are allowed during this exam.**
3. **You may use one page of notes, but no books or other assistance during this exam.**
4. **Read each question carefully, and answer each question completely.**
5. **Write your solutions clearly in your Blue Book.**
   (a) **Carefully indicate the number and letter of each question and question part.**
   (b) **Present your answers in the same order they appear in the exam.**
   (c) **Start each problem on a new side of a page.**
6. **Show all of your work. No credit will be given for unsupported answers, even if correct.**
7. **Write Name & PID on this exam sheet and return it inside the front cover of your Blue Book.**

---

0. (2 points) Carefully read and complete the instructions at the top of this exam sheet and any additional instructions written on the chalkboard during the exam.

1. Consider the linear system

$$\begin{array}{rcrcrcl} x_1 & + & 3x_2 & - & 3x_3 & = & 7 \\ & & x_2 & - & 4x_3 & = & 5 \end{array}$$

   (a) (4 points) Determine the solution set and write it in parametric form.

   (b) (2 points) Write the parametric form for the solution set of the corresponding homogeneous equation.

2. $A = \begin{bmatrix} 2 & 3 & 1 & 8 & 2 & 3 \\ -1 & 2 & 3 & 3 & 3 & 3 \\ 3 & 1 & -2 & 5 & 3 & 8 \\ 1 & 4 & 3 & 9 & 2 & 1 \end{bmatrix}$ is row equivalent to $B = \begin{bmatrix} 1 & 0 & -1 & 1 & 0 & 1 \\ 0 & 1 & 1 & 2 & 0 & -1 \\ 0 & 0 & 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$.

   Find a basis for each of the following subspaces associated with the matrix A:

   (a) (1 point) $\mathrm{Col}(A)$, the column space of A.

   (b) (2 points) $\mathrm{Col}\left(A^T\right)$, the column space of A^T.

   (c) (3 points) $\mathrm{Row}(A)^{\perp}$, the orthogonal complement of the row space of A.

3. A certain 6×11 matrix A has rank 3; that is, $\mathrm{rank}(A) = 3$. Determine each of the following quantities. Be sure to (briefly!) indicate how you arrived at your answers.

   (a) (2 points) $\dim [\mathrm{Nul}\,(A)]$

   (b) (2 points) $\dim \left[\mathrm{Col}\left(A^T\right)\right]$

   (c) (2 points) $\dim \left[\mathrm{Nul}\left(A^T\right)\right]$

Exam continues with Problems 4 – 8 on the other side of this sheet.

(This exam is worth 50 points.)

4. Let A be a *nonzero* $n \times n$ matrix such that $A^2 = 0$, the $n \times n$ zero matrix.

   (a) (2 points) Show that $\lambda = 0$ is the only eigenvalue of A.

   (b) (4 points) Show that A is *not* diagonalizable.

5. Let A be an invertible matrix.

   (a) (2 points) Explain why $\lambda = 0$ cannot be an eigenvalue of A.

   (b) (4 points) Show that if $\mathbf{w}$ is an eigenvector of A corresponding to λ, then $\mathbf{w}$ is also an eigenvector of A^{-1} corresponding to $\frac{1}{\lambda}$.

6. (6 points) Solve the initial value problem $\mathbf{y}'(t) = A\,\mathbf{y}(t)$; $\mathbf{y}(0) = \mathbf{c}$, where

$$A = \begin{bmatrix} 6 & 2 \\ 2 & 3 \end{bmatrix}; \quad \mathbf{c} = \begin{bmatrix} 3 \\ 4 \end{bmatrix}.$$

7. Given the matrix $A = \begin{bmatrix} 1 & -1 & 3 \\ 1 & 3 & -2 \\ 1 & 3 & 1 \\ 1 & -1 & 0 \end{bmatrix}$.

   (a) (4 points) Find an orthonormal basis for the column space of A.

   (b) (2 points) Factor A into a product QR, where Q is a matrix with orthonormal columns and R is an upper triangular matrix.

8. Consider the following system of linear equations.

$$\begin{array}{rcrcr} x_1 & - & x_2 & = & 1 \\ -2x_1 & + & x_2 & = & -2 \\ x_1 & + & 2x_2 & = & 4 \end{array}$$

   (a) (2 points) Explain why the system is inconsistent.

   (b) (4 points) Find the least-squares solution to the system.