Math 20F **Midterm Exam 1** **Name:** ____________

January 27, 2015 **Version A** **PID:** ____________

Instructions

1. **Write your *Name*, *PID*, *Section*, and *Exam Version* on the front of your Blue Book.**
2. **No calculators or other electronic devices are allowed during this exam.**
3. **You may use one page of notes, but no books or other assistance during this exam.**
4. **Read each question carefully, and answer each question completely.**
5. **Write your solutions clearly in your Blue Book.**
   (a) **Carefully indicate the number and letter of each question and question part.**
   (b) **Present your answers in the same order they appear in the exam.**
   (c) **Start each problem on a new side of a page.**
6. **Show all of your work. No credit will be given for unsupported answers, even if correct.**
7. **Write Name & PID on this exam sheet and return it inside the front cover of your Blue Book.**

---

0. (1 point) Carefully read and complete the instructions at the top of this exam sheet and any additional instructions written on the chalkboard during the exam.

1. Let S be the linear system $\left\{\begin{array}{rcrcrcr} x_1 & - & 2x_2 & + & 2x_3 & = & 5 \\ & & x_2 & + & 3x_3 & = & 2 \\ x_1 & - & 3x_2 & - & x_3 & = & 3 \end{array}\right.$.

   (a) (4 points) Find the solution set of the linear system S and write it in parametric form.

   (b) (2 points) Write the solution set of the corresponding homogoneous linear system. (Hint: This should not require any computation.)

2. Let $A = \begin{bmatrix} 1 & 3 & 5 \\ 3 & 1 & 7 \\ 1 & -1 & 1 \end{bmatrix}$. The reduced echelon form of A is $\begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{bmatrix}$.

   (a) (3 points) Find the solution set of the homogeneous equation $A\mathbf{x} = \mathbf{0}$ and write it in parametric form.

   (b) (3 points) If possible, find a vector $\mathbf{b} \in \mathbb{R}^3$ that *cannot* be written as a linear combination of the columns of A. If it is not possible, explain why it is not possible.

3. Let $S = \left\{ \begin{bmatrix} 1 \\ 2 \\ -2 \end{bmatrix}, \begin{bmatrix} -2 \\ 1 \\ 2 \end{bmatrix}, \begin{bmatrix} -3 \\ 4 \\ 2 \end{bmatrix} \right\}$.

   (a) (3 points) Determine whether S is linearly dependent or linearly independent.

   (b) (3 points) Determine whether or not S spans $\mathbb{R}^3$.

4. (6 points) Let $T : \mathbb{R}^3 \to \mathbb{R}^3$ be a linear transformation such that

$$T\left(\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}\right) = \begin{bmatrix} 2 \\ 0 \\ 0 \end{bmatrix}, \qquad T\left(\begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}\right) = \begin{bmatrix} 0 \\ 2 \\ 0 \end{bmatrix}, \qquad T\left(\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}\right) = \begin{bmatrix} 0 \\ 0 \\ 2 \end{bmatrix}.$$

   Find the standard matrix for T.

(This exam is worth 25 points.)