Math 20F | **Midterm Exam 2** | **Name:** ______________
February 24, 2015 | **Version A** | **PID:** ______________

Instructions

1. **Write your *Name*, *PID*, *Section*, and *Exam Version* on the front of your Blue Book.**
2. **No calculators or other electronic devices are allowed during this exam.**
3. **You may use one page of notes, but no books or other assistance during this exam.**
4. **Read each question carefully, and answer each question completely.**
5. **Write your solutions clearly in your Blue Book.**
   (a) **Carefully indicate the number and letter of each question and question part.**
   (b) **Present your answers in the same order they appear in the exam.**
   (c) **Start each problem on a new side of a page.**
6. **Show all of your work. No credit will be given for unsupported answers, even if correct.**
7. **Write Name & PID on this exam sheet and return it inside the front cover of your Blue Book.**

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0. (1 point) Carefully read and complete the instructions at the top of this exam sheet and any additional instructions written on the chalkboard during the exam.

1. $A = \begin{bmatrix} 1 & -4 & -3 & 1 & -2 & -8 \\ 3 & 2 & 5 & 1 & 6 & 2 \\ 2 & 3 & 5 & 2 & 7 & 6 \\ 4 & -1 & 3 & 2 & 5 & -4 \end{bmatrix}$ is row equivalent to $B = \begin{bmatrix} 1 & 0 & 1 & 0 & 1 & -1 \\ 0 & 1 & 1 & 0 & 1 & 2 \\ 0 & 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$.

   Find a basis for each of the following subspaces associated with the matrix A:

   (a) (1 point) $\mathrm{Col}(A)$, the column space of A.
   (b) (2 points) $\mathrm{Col}\left(A^T\right)$, the column space of A^T.
   (c) (3 points) $\mathrm{Nul}(A)$, the null space of A.

2. (6 points) The set $\mathcal{S} = \left\{ \begin{bmatrix} 1 \\ 2 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 2 \\ 2 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 0 \\ 1 \end{bmatrix} \right\}$ is a linearly independent subset of $\mathbb{R}^4$. Extend $\mathcal{S}$ to a basis of $\mathbb{R}^4$; that is, find a vector $\mathbf{b}$ so that $\mathcal{B} = \mathcal{S} \cup \{\mathbf{b}\}$ is a basis for $\mathbb{R}^4$. Be sure to explain how you know that the set $\mathcal{B}$ you obtain is a basis for $\mathbb{R}^4$.

3. (6 points) Let A be a $m \times n$ matrix and B a $n \times p$ matrix. Suppose that the $m \times p$ matrix $C = AB$ has the property that $C\mathbf{x} = \mathbf{b}$ has at least one solution for each $\mathbf{b}$ in $\mathbb{R}^m$. Find $\dim[\mathrm{Col}(A)]$, the dimension of the column space of A. Be sure to justify your answer.

4. (6 points) Let $\mathscr{E}$ be a homogeneous system of 8 linear equations in 10 unknowns. Is it possible for the solution set of $\mathscr{E}$ to consist of all multiples of a fixed nonzero solution? Justify your answer.

(This exam is worth 25 points.)