Checklist of final topics to review (Math 20C)

December 4, 2019

Page references refer to Vector Calculus (6th Ed.) - Marsden and Tromba

Chapter 1 (Assumed knowledge but not a focus)

Sections 2.1-2.5 (Assumed knowledge but not a focus)

Chapter 2 - Differentiation

§2.6 - Gradients and Directional Derivatives

☐ Directional derivatives - (page 135, 136)
  ☐ Definition
  ☐ Calculation (using gradients)
  ☐ Geometric interpretation (rate of change/slope of a function at a point in a given direction)

☐ Direction of fastest increase of a function (Theorem 13) - (page 137, 138)
  ☐ Finding direction of greatest rate of change - example 5 (page 138)

☐ The gradient of normal to level surfaces (Theorem 14) - (page 138)
  ☐ Finding normal direction to a surface (and more generally level set of a function)

☐ Tangent plane (to a level surface $f(x, y, z) = k$) - (page 139)
Chapter 3 - Higher Order Derivatives: Maxima and Minima

§3.1 - Iterated Partial Integrals
- Notation and Definition - (page 150)
- Examples
  - Examples 1-4 (Basic Calculations) - (page 150-152)
  - Example 5 (A more abstract use of the formulae) - (page 153)
- Theorem 1 (Equality of mixed partials) - (page 151)

§3.2 - Taylor’s Theorem
- Theorems 2 and 3 (First and Second Order Formulae) - (page 160)
- Examples
  - Examples 1,2 (Basic 2nd order calculation) - (page 163)
  - Examples 3 (1st and 2nd order calculation - compare the two!) - (page 164)
  - Example 4 (Application: Quadratic approximation) - (page 164,165)

§3.3 - Extrema of real valued functions
- Terminology and basic (formal) definitions (local min/max, saddle point) - (page 168)
- Theorem 4 (1st derivative test) - (page 169)
- You can skip the following subsections (bottom of pg 171 - 176):
  - Second-Derivative Test for Local Extrema,
  - Determinant Test for Positive Definiteness,
  - General Second-Derivative Tests (n-variables)
- Theorem 6 (Second-Derivative Maximum-Minimum Test for Functions of Two Variables) - (page 176)
- Examples 6, 7, 8, 9 - (page 176 - 179)
- Definitions and terminology (global maxima and minima) - (page 180)
- Theorem 7 (Global Existence Theorem for Maxima and Minima) - (page 180)
- Strategy for Finding the Absolute Maxima and Minima on a Region with Boundary - (page 181)
Example 11 (Compare this solution with one using the method of Lagrange multiplier instead, which one do you prefer?)

§3.4 - Constrained Extrema and Lagrange Multipliers

- Geometric meaning of constrained optimization - (page 185)
- Theorem 8 (The Method of Lagrange Multipliers) - (page 186)
- Examples 1 - 4 - (page 188-190)
- Several Constraints - (page 191)
- Example 5 (problem with 2 constraints)
- Examples 6-8 (Using Lagrange method to help solve absolute max/min type problems from §3.3) - (page 192-195)
Chapter 4 - Vector Valued Functions

§4.1 - Acceleration and Newtons Second Law
   □ Basic definitions (velocity, speed, acceleration)

§4.2 - Arc Length
   □ Definition - (page 228)
   □ Examples 1 - 4 - (page 228-230)
Chapter 5 - Double and Triple Integrals

§5.1 - Introduction

☐ Volume definition of double integral - (page 263, 264)

☐ Basic examples - that do not require integration (example 1 - constant functions, triangular prisms) - (page 264)

☐ Reducing double integrals (over rectangular $R$) to iterated integrals - (page 267)

☐ Geometric interpretation (Cavalieri’s Principle) - (page 265-267)

§5.2 - The Double Integral over a Rectangle

☐ Limit definition of double integral - (page 272)

☐ Basic properties of double integrals (linearity, homogeneity, monotonicity, additivity) - (page 275)

☐ Fubini’s Theorem (Theorem 3) - (page 277)

☐ Application to problem solving: Order of integration over rectangles does not matter - examples 1, 2, 3 (page 279-280)

§5.3 - The Double Integral Over More General Regions

☐ Elementary regions - (page 283)

☐ Rewriting a region in $\mathbb{R}^2$:

☐ As $y$-simple

☐ As $x$-simple

☐ Reducing double integrals (over general $R$) to iterated integrals (Theorem 4 and 4’) (page 286, 287)

§5.4 - Changing the Order of Integration

☐ Rewriting $y$-simple regions as $x$-simple (and vice versa)

☐ Changing order of integration - (page 289-290)

☐ Evaluating difficult (or impossible) integrals by changing order of integration - examples 1 and 2 (page 290-291)

☐ Example 1 - (page 290)

$$f(x, y) = (a^2 - y^2)^{1/2}$$

(Easier to integrate wrt $x$ first)
Example 2 - (page 291)

\[ f(x, y) = (x - 1)\sqrt{1 + e^{2y}} \]

(Easier to integrate wrt \( x \) first)

Example 3 - (Lecture)

\[ f(x, y) = xe^{y^3} \]

(Easier to integrate wrt \( x \) first)

Example 4 - (Lecture)

\[ f(x, y) = \sin(y^3) \]

(Easier to integrate wrt \( x \) first)

§5.5 - Triple Integrals

Definition of a triple integral (limiting case of an approximation) - (page 294-295)

Reduction of a triple integral to iterated integrals over a box \( B \) - (page 295-296)

Example 1 - (page 296)

Example 2 - (page 297)

Elementary regions in space - (page 297)

Rewriting regions in \( \mathbb{R}^3 \) as an elementary region

Example 3 - (page 297, 298)

Example 5 - (page 300, 301)

Reduction of a triple integral to iterated integrals over an elementary region - (page 298)

Example 4 - (page 298, 299)

Example 5 - (page 300, 301)

Finding volume of a solid (by integrating 1)

Example 4 - (page 298, 299)