Math 20D - Fall 2008 - Midterm I

Name: ______________________________

Student ID: ___________________________

Section time: __________________________

Instructions:

Please print your name, student ID and section time.

During the test, you may not use books, calculators or telephones. You may use a "cheat sheet" of notes which should be at most half a page, front and back.

Read each question carefully, and show all your work. Answers with no explanation will receive no credit, even if they are correct.

There are 6 questions which are worth 60 points. You have 50 minutes to complete the test.

Question	Score	Maximum
1		12
2		10
3		10
4		10
5		12
6		6
Total		60

Problem 1. [*12 points.*]

Consider the linear first order equation

$$t^2 y' + 3ty = 2e^{t^2}.$$

(i) [*4 points.*] Compute an integrating factor for the differential equation.

(ii) [*4 points.*] Find the general solution.

(iii) [*4 points.*] Find the solution which satisfies the initial condition $y(1) = 0$. What is the maximal interval where the solution is defined?

Problem 2. [*10 points.*]

A tank originally contains 10 gallons of fresh water. Water containing 3 lb of salt per gallon is poured into the tank at a rate of 2 gal/min. The mixture is allowed to leave the tank at the same rate.

(i) [*5 points.*] Write down the differential equation for the amount $Q(t)$ of salt in the tank at time t.

(ii) [*5 points.*] Find the amount of salt in the tank after 10 minutes.

Problem 3. [*10 points.*]

Consider the differential equation

$$(3x^2 + y^2) + (2xy + 1)y' = 0.$$

(i) [*4 points.*] Explain why the differential equation is exact.

(ii) [*6 points.*] Solve the differential equation. It suffices to give the solution implicitly.

Problem 4. [*10 points.*]

Consider the autonomous equation

$$\frac{dy}{dt} = 4y - y^2.$$

(i) [*7 points.*] Determine the critical points and indicate their type i.e. asymptotically stable, unstable, semistable. Sketch the phase line.

(ii) [*3 points.*] What is the long-term behavior of the solution satisfying the initial value $y(0) = 2$?

Problem 5. [*10 points.*]

Find the general solution of the differential equation $y'' + 4y' + 13y = 0$.

Problem 6. [*8 points.*]

Consider the differential equation

$$y'' + 2ty' + q(t)\, y = 0,$$

for some unknown function $q(t)$.

Two solutions y_1 and y_2 of the differential equation satisfy the initial conditions

$$y_1(0) = 1, \;\; y_2(0) = 2$$
$$y_1'(0) = -1, \;\; y_2'(0) = 3.$$

(i) [*4 points*] Determine the Wronskian $W(y_1, y_2)$ as a function of t. Do y_1 and y_2 form a fundamental pair of solutions?

(ii) [*4 points*] A third solution satisfies the initial value problem

$$y(0) = 1, y'(0) = 7.$$

Express this solution in terms of y_1 and y_2.