Math 20D - Fall 2008 - Midterm II

Name: ____________________

Student ID: ____________________

Section time: ____________________

Instructions:

Please print your name, student ID and section time.

During the test, you may not use books, calculators or telephones. You may use a "cheat sheet" of notes which should be at most one page (front).

Read each question carefully, and show all your work. Answers with no explanation will receive no credit, even if they are correct.

There are 4 questions which are worth 60 points. You have 50 minutes to complete the test.

Question	Score	Maximum
1		10
2		15
3		15
4		20
Total		60

Problem 1. [*10 points.*]

Solve the following differential equation by variation of parameters

$$y'' - 2y' + y = t^{-2}e^t.$$

Problem 2. [*15 points.*]

Using undetermined coefficients, find a particular solution of the differential equation

$$y'' - 8y' + 7y = -e^{2t}(5t + 9).$$

Problem 3. [*15 points.*]

(i) [*4 points*] Consider the system

$$\mathbf{x}' = \begin{bmatrix} 3 & 4 \\ -4 & 3 \end{bmatrix} \mathbf{x}.$$

An eigenvalue of the matrix of coefficients is

$$\lambda = 3 + 4i.$$

(*You do not need to check this fact.*) Without solving the system, carefully sketch the trajectory of the solution which has the initial value $\mathbf{x}(0) = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$. Be sure to include arrows on your diagram that indicate the direction of the trajectory.

(ii) [*5 points*] The defective matrix $A = \begin{bmatrix} 3 & 1 \\ -1 & 1 \end{bmatrix}$ has a *repeated* eigenvalue $\lambda = 2$ and a corresponding eigenvector $\mathbf{v} = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$. (*You do not need to check this fact.*) Write down two independent solutions $\mathbf{x}_1$ and $\mathbf{x}_2$ for the system $\mathbf{x}' = A\mathbf{x}$.

(iii) [*6 points*] The general solution of a certain first order system of differential equations is

$$\mathbf{x} = C_1 e^{2t} \begin{bmatrix} 1 \\ 1 \end{bmatrix} + C_2 e^{3t} \begin{bmatrix} -1 \\ 2 \end{bmatrix}.$$

Carefully sketch the corresponding trajectories.

Problem 4. [*20 points.*]

Consider the system $\mathbf{x}' = A\mathbf{x}$ where

$$A = \begin{bmatrix} -3 & -2 \\ 4 & 3 \end{bmatrix}.$$

(i) [*6 points*] Write down the general solution.

(ii) [*4 points*] Compute the matrix exponential e^{At}.

(iii) [*2 points*] Solve the initial value problem $\mathbf{x}(0) = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$.

(iv) [*8 points*] Using variation of parameters, find a particular solution for

$$\mathbf{x}' = A\mathbf{x} + \begin{bmatrix} e^{2t} \\ 0 \end{bmatrix}.$$