

Math 20D - Fall 2011 - Midterm II

Name: \_\_\_\_\_

Student ID: \_\_\_\_\_

Section time: \_\_\_\_\_

**Instructions:**

Please print your name, student ID and section time.

During the test, you may not use books, calculators or telephones. You may use a "cheat sheet" of notes which should be at most half a page, front and back.

Read each question carefully, and show all your work. Answers with no explanation will receive no credit, even if they are correct.

There are 4 questions which are worth 45 points. You have 50 minutes to complete the test.

| Question | Score | Maximum |
|----------|-------|---------|
| 1        |       | 10      |
| 2        |       | 8       |
| 3        |       | 11      |
| 4        |       | 16      |
| Total    |       | 45      |

**Problem 1.** [10 points; 5, 5.]

The matrix  $A = \begin{bmatrix} 3 & 1 \\ -1 & 5 \end{bmatrix}$  has a repeated eigenvalue  $\lambda = 4$  and an eigenvector  $\vec{v} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$ .  
(You do not need to check this fact.)

(i) Find a fundamental pair of solutions for the system  $\vec{x}' = A\vec{x}$ .

(ii) Calculate the matrix exponential  $e^{At}$ .

**Problem 2.** [8 points.]

Using undetermined coefficients, find a particular solution to the differential equation:

$$y'' - 2y' - 3y = 3 - 10 \sin t.$$

**Problem 3.** [11 points.]

Using variation of parameters, find a particular solution to the differential equation:

$$y'' - 2y' + 2y = e^t \sin t \cos t.$$

Please write the solution in simplest form.

**Problem 4.** [16 points; 6, 4, 6.]

Consider the system

$$\vec{x}' = \begin{bmatrix} 1 & 1 \\ 4 & -2 \end{bmatrix} \vec{x}.$$

(i) Write down the general solution.

(ii) Sketch the trajectories of the solutions and indicate their type.

(iii) Using variation of parameters, find a particular solution to the inhomogeneous system

$$\vec{x}' = \begin{bmatrix} 1 & 1 \\ 4 & -2 \end{bmatrix} \vec{x} + \begin{bmatrix} 5e^t \\ 0 \end{bmatrix}$$