Mathematics 20D
First Midterm, October 27, 2014

Maximum score on this midterm is 50 points.

Write your solutions in the space provided. In case you need more space than provided use the back-page.

Important: Show your reasoning! Answers without explanations receive no credit, even if they are correct!

Full name: ____________________

Section: ____________________

Student ID: ____________________

Signature: ____________________

Problem 1. __________

Problem 2. __________

Problem 3. __________

Problem 4. __________

Problem 5. __________

Total: ____________________

Problem 1. (10 points: 7+2+1) i) Solve the following differential equation:

$$ty' + 2y = \frac{\cos t}{t}, \qquad y(\pi) = 0.$$

ii) Compute $\lim_{t\to\infty} y(t)$.

iii) What is the maximal time interval on which the solution is defined?

Problem 2. (10 points) The size of a colony of bacteria $y(t)$ grows according to the differential equation:

$$\frac{dy}{dt} = y^2 - 5y + 6.$$

Determine the critical points, the equilibrium solutions and indicate their type, that is stable, unstable or semi-stable. What is the long time behavior of the solution with initial data $y(0) = 1$? (here discuss whether $y(t)$ is increasing/decreasing and indicate $\lim_{t\to\infty} y(t)$; no need to discuss the concavity).

Problem 3. i) (10 points= 7+2+1)Solve the following differential equation:

$$4y'' - y = 0, \qquad y(0) = 2, y'(0) = \beta.$$

ii) Find β so that the solution approaches zero as $t \to \infty$.

iii) Find β so that the solution approaches zero as $t \to -\infty$.

Problem 4. (8 points: 6+2) Consider the differential equation:

$$t^2y'' - 3ty' + 3y = 0, \qquad t > 0.$$

i) Check that $y_1(t) = t$ and $y_2(t) = t^3$ form a fundamental set of solutions (that includes checking that they are solutions!).

ii) What is the general solution of the above equation?

Problem 5. (12 points=2+10) i) Consider the differential equation:

$$(3xy + y^2) + (x^2 + xy)y' = 0$$

Is this equation exact?

ii) Solve the equation. Leave the solution in implicit form. HINT: if the answer in part i) is negative, then seek an integrating factor $\mu = \mu(x)$ that makes the equation exact.