Math 20D **Name:** _____

October 22, 2018 **Midterm** **PID:** _____

Instructions

1. **Write your *Name* and *PID* on the front of your Blue Book.**
2. **No calculators or other electronic devices are allowed during this exam.**
3. **You may use a double sided page of notes.**
4. **Write your solutions clearly in your Blue Book.**
   (a) **Carefully indicate the number and letter of each question and question part.**
   (b) **Present your answers in the same order as they appear in the exam.**
   (c) **Start each numbered problem on a new side of a page.**
5. **Show all of your work and justify all your claims. No credit will be given for unsupported answers, even if correct.**

Complete 5 out of the 6 questions

1. (10 points) (a) Solve the initial value problem

$$y' - y = e^{-t}, \quad y(0) = y_0$$

(b) Explain how the behaviour of the solution $y(t)$ as $t \longrightarrow \infty$ depends on the value of the initial value y_0.

Hint: There are 3 different cases depending on y_0.

2. (10 points) Find the general solution to the following differential equations

(a)
$$\frac{d^2y}{dt^2} + 6\frac{dy}{dt} + 9y = 0$$

(b)
$$4\frac{d^2y}{dt^2} + 4\frac{dy}{dt} - 3y = 0$$

(c)
$$\frac{d^2y}{dt^2} - 2\frac{dy}{dt} + 5y = 0$$

3. (10 points) (a) Find the explicit solution to the initial value problem

$$e^t - yy' = 0, \quad y(0) = 1$$

(b) For which values of t is this solution valid?

4. (10 points) Find the general solution to the differential equation

$$\frac{dy}{dx} = \frac{x - y - 1}{x + y + 5}$$

5. (10 points) Consider the following nonhomogeneous equation

$$\frac{d^2y}{dt^2} + \frac{dy}{dt} - 2y = f(t)$$

Determine the form of a particular solution in the following cases (**you do not have to calculate any unknown coefficients**)

(a) $f(t) = t^2 - 1$

(b) $f(t) = e^{2t}$

(c) $f(t) = \cos(2t)$

(d) $f(t) = te^t$

6. (10 points) Solve the initial value problem

$$4\frac{d^2x}{dt^2} - 4\frac{dx}{dt} + x = 0; \quad x(0) = 2, \quad x'(0) = 2$$