
Instructions

1. Write your *Name* and *PID* on the front of your Blue Book.
 2. No calculators or other electronic devices are allowed during this exam.
 3. You may use a double sided page of notes.
 4. Write your solutions clearly in your Blue Book.
 - (a) Carefully indicate the number and letter of each question and question part.
 - (b) Present your answers in the same order as they appear in the exam.
 - (c) Start each numbered problem on a new side of a page.
 5. Show all of your work and justify all your claims. No credit will be given for unsupported answers, even if correct.
-

Complete 5 out of the 6 questions

1. (10 points) Solve the initial value problem

$$x^2 y''(x) = 5xy'(x) - 13y(x), \quad y(-1) = 1, \quad y'(-1) = -1 \quad (1)$$

2. (10 points) Find the general solution to the following differential equation

$$y'' + 16y = \tan(4t) \quad (2)$$

3. (10 points) Solve the initial value problem

$$\mathbf{x}' = \begin{pmatrix} 1 & 1 & 0 \\ 0 & 2 & 0 \\ 0 & -1 & 4 \end{pmatrix} \mathbf{x}, \quad \mathbf{x}(0) = \begin{pmatrix} -3 \\ -2 \\ 1 \end{pmatrix} \quad (3)$$

4. (10 points) Find the general solution to the non-homogeneous system

$$\mathbf{x}' = \begin{pmatrix} 3 & -2 \\ 4 & -1 \end{pmatrix} \mathbf{x} + \begin{pmatrix} -6e^{-t} \\ -4e^{-t} \end{pmatrix} \quad (4)$$

5. (10 points) (a) Verify that

$$\left\{ \begin{pmatrix} 1 \\ 1 \end{pmatrix} e^t, \begin{pmatrix} 1 \\ 3 \end{pmatrix} e^{-t} \right\}$$

is a fundamental solution set to the homogeneous system

$$\mathbf{x}' = \begin{pmatrix} 2 & -1 \\ 3 & -2 \end{pmatrix} \mathbf{x} \quad (5)$$

- (b) Using variation of parameters, find a particular solution to the non-homogeneous system

$$\mathbf{x}' = \begin{pmatrix} 2 & -1 \\ 3 & -2 \end{pmatrix} \mathbf{x} + \begin{pmatrix} e^t \\ t \end{pmatrix}$$

6. (10 points) Find a fundamental matrix for the system

$$\mathbf{x}' = \begin{pmatrix} 0 & 1 & 4 \\ 0 & 0 & 2 \\ 0 & 0 & 0 \end{pmatrix} \mathbf{x} \quad (6)$$