

Math 20D - Spring 2017 - Midterm I

Name: _____

Student ID: _____

Section time: _____

Instructions:

Please print your name, student ID and section time.

During the test, you may not use books, calculators or telephones. You may use a "cheat sheet" of notes which should be at most a page, front only.

Read each question carefully, and show all your work. Answers with no explanation will receive no credit, even if they are correct.

There are 5 questions which are worth 50 points. You have 50 minutes to complete the test.

Question	Score	Maximum
1		10
2		10
3		10
4		10
5		10
Total		50

Problem 1. [10 points.]

The differential equation

$$(2x \sin y + y^2) + (x^2 \cos y + 2xy + e^y)y' = 0, \quad y(0) = 0$$

is exact (**you do not need to check this**). Solve this initial value problem.

(You can leave the answer in implicit form, without solving explicitly for y .)

Problem 2. [10 points.]

Consider the autonomous differential equation

$$\frac{dy}{dt} = y^2 - 4y + 3.$$

Determine the critical points and indicate their type i.e. asymptotically stable, unstable, semistable.

What is the long-term behavior of the solution satisfying the initial value $y(0) = 2$?

Problem 3. [10 points.]

Find the solution to the initial value problem

$$t^3 y' + 5t^2 y = 3, \quad y(-1) = 2.$$

What is the largest interval over which the solution is defined?

Problem 4. [10 points.]

Write down the general solution of the differential equation

$$y'' + 4y' + 5y = 0$$

and sketch its graph.

Problem 5. [*10 points.*]

A swimming pool originally contains 200 gallons of water and 50lb of salt. Water containing 4 lb of salt per gallon is poured into the pool at a rate of 2 gal/min. The mixture is simultaneously allowed to leave the pool at the same rate. Find the amount of salt in the pool at time t .