

Math 20D - Spring 2017 - Midterm II

Name: _____

Student ID: _____

Section time: _____

Instructions:

Please print your name, student ID and section time.

During the test, you may not use books, calculators or telephones. You may use a "cheat sheet" of notes which should be at most a page, front only.

Read each question carefully, and show all your work. Answers with no explanation will receive no credit, even if they are correct.

There are 4 questions which are worth 40 points. You have 50 minutes to complete the test.

Question	Score	Maximum
1		10
2		10
3		10
4		10
Total		40

Problem 1. [10 points.]

Using undetermined coefficients, find a particular solution for the differential equation

$$y'' - y' - 2y = 4e^{3t} + 5 \sin t.$$

Problem 2. [10 points.]

Using variation of parameters, find a particular solution for the differential equation

$$y'' - 6y' + 9y = \frac{e^{3t}}{t+1}.$$

Problem 3. [10 points; 3, 2, 5.]

Consider the linear system $\vec{x}' = A\vec{x}$. It is known that the matrix A has eigenvalues $\lambda = 2$ with eigenvector $\vec{v}_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$ and $\lambda_2 = 4$ with eigenvector $\vec{v}_2 = \begin{bmatrix} 3 \\ 2 \end{bmatrix}$.

(i) Write down a pair \vec{x}_1, \vec{x}_2 of fundamental solutions and verify that $W(\vec{x}_1, \vec{x}_2) \neq 0$.

(ii) Write down the general solution of the system.

(iii) Sketch a few of the trajectories and classify the type of critical point at the origin.

Problem 4. [10 points; 6, 4.]

Consider the linear system

$$\vec{x}' = A\vec{x}, \quad A = \begin{bmatrix} -1 & 4 \\ -2 & 3 \end{bmatrix}.$$

(i) Write down the general solution.

(ii) Sketch the trajectory of the solution which satisfies $\vec{x}(0) = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$. Clearly indicate the direction of the trajectory, and type of critical point at the origin.