**Math 20D** **Name:** _____

**September 8, 2018** **Final** **PID:** _____

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**Instructions**

1. **Write your *Name* and *PID* on the front of your Blue Book.**
2. **No calculators or other electronic devices are allowed during this exam.**
3. **You may use a double sided page of notes.**
4. **Write your solutions clearly in your Blue Book.**
   (a) **Carefully indicate the number and letter of each question and question part.**
   (b) **Present your answers in the same order as they appear in the exam.**
   (c) **Start each numbered problem on a new side of a page.**
5. **Show all of your work and justify all your claims. No credit will be given for unsupported answers, even if correct.**

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**Complete 10 out of the 11 questions**

1. (10 points) Solve the initial value problem:

$$\mathbf{x}' = \begin{pmatrix} 2 & 1 \\ 2 & 3 \end{pmatrix} \mathbf{x}, \quad \mathbf{x}(0) = \begin{pmatrix} -1 \\ -8 \end{pmatrix}$$

2. (10 points) Solve the equation

$$(y^3 + y\cos(x))dx + (3y^2x + \sin(x))dy = 0$$

3. (10 points) Find the general solution to the following homogeneous equations:

   (a)
$$\frac{d^2y}{dx^2} - 3\frac{dy}{dx} + 2y = 0$$

   (b)
$$\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + 2y = 0$$

   (c)
$$4\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + y = 0$$

4. (10 points) Find the general solution to the system:

$$\mathbf{x}' = \begin{pmatrix} 2 & 0 & 0 \\ 1 & -2 & 1 \\ 1 & 1 & -2 \end{pmatrix} \mathbf{x}$$

5. (10 points) Solve the initial value problem

$$e^{2y}\frac{dy}{dx} - 8x^3 = 0, \quad y(1) = 0$$

6. (10 points) Find the general solution to the equation:

$$y''(t) - \frac{1}{t}y'(t) + \frac{5}{t^2}y(t) = 0, \quad t < 0$$

   How would your answer change if we wanted a solution valid for $t > 0$?

7. (10 points) Consider the following differential equation

$$y''(t) + 2y'(t) + y(t) = f(t)$$

Using the method of undetermined coefficients, determine the general form of a particular solution $y_p(t)$ in the following cases (**do not calculate the unknown constants**):

(a) $f(t) = t^3 - t$

(b) $f(t) = t(e^t - e^{-t})$

(c) $f(t) = \cos(2t) - 1$

(d) $f(t) = \cos(t)e^{-t}$

8. (10 points) Find a general solution to the system of differential equations

$$\frac{dx}{dt} = 4x(t) + 4y(t)$$
$$\frac{dy}{dt} = -2x(t)$$

9. (10 points) (a) Verify that $\left\{ \begin{pmatrix} e^{7t} \\ 2e^{7t} \end{pmatrix}, \begin{pmatrix} e^{-5t} \\ -2e^{-5t} \end{pmatrix} \right\}$ is a fundamental solution set to the system

$$\mathbf{x}' = \begin{pmatrix} 1 & 3 \\ 12 & 1 \end{pmatrix} \mathbf{x}$$

(b) Solve the initial value problem

$$\mathbf{x}' = \begin{pmatrix} 1 & 3 \\ 12 & 1 \end{pmatrix} \mathbf{x}, \quad \mathbf{x}(0) = \begin{pmatrix} 4 \\ 0 \end{pmatrix}$$

10. (10 points) Using variation of parameters, find a particular solution to the differential equation

$$y'' - 2y' + y = \frac{e^t}{t}$$

11. (10 points) Find a particular solution for the system:

$$\mathbf{x}' = \begin{pmatrix} 3 & 1 \\ -1 & 0 \end{pmatrix} \mathbf{x} + \begin{pmatrix} -1 - 4t \\ 2 + t \end{pmatrix}$$