Math 20D **Name:** _____

August 23, 2018 **Midterm** **PID:** _____

Instructions

1. **Write your *Name* and *PID* on the front of your Blue Book.**
2. **No calculators or other electronic devices are allowed during this exam.**
3. **You may use a double sided page of notes.**
4. **Write your solutions clearly in your Blue Book.**
   (a) **Carefully indicate the number and letter of each question and question part.**
   (b) **Present your answers in the same order as they appear in the exam.**
   (c) **Start each numbered problem on a new side of a page.**
5. **Show all of your work and justify all your claims. No credit will be given for unsupported answers, even if correct.**

Complete 5 out of the 6 questions

1. (10 points) Find the general solution to the differential equation

$$\frac{dy}{dx} = \frac{y}{x} + x^2\cos(x)$$

2. (10 points) Solve the initial value problem

$$\frac{dy}{dx} = (x+y)^2 - (x-y)^2, \quad y(1) = e^2$$

Note: When simplifying you can assume $y(x) > 0$ for all x

3. (10 points) **[This question has multiple parts]**

   (a) Find the general solution $y_h(t)$ to the homogeneous differential equation

$$\frac{d^2y}{dt^2} + 6\frac{dy}{dt} + 9y = 0$$

   (b) Give the general form of a **particular solution** $y_p(t)$ to

$$\frac{d^2y}{dt^2} + 6\frac{dy}{dt} + 9y = te^t \tag{1}$$

   (You do not need to solve for any unknown constants)

   (c) Using (a) and (b) give the general solution to the non-homogeneous equation (1)

4. (10 points) Find the general solution to the differential equation

$$\frac{dy}{dx} = \frac{2x-y}{x+y-4}$$

Hint: It may be a good idea to rewrite this as an equation involving a differential form.

5. (10 points) Solve the initial value problem

$$y''(t) + 4y(t) = 4\sin(2t); \quad y(0) = 1,\ y'(0) = 3 \tag{2}$$

Hint: The general form of the particular solution to (2) is given by

$$y_p(t) = At\cos(2t)$$

6. (10 points) Solve the initial value problem for the **Cauchy-Euler equation**

$$t^2y''(t) + 7ty'(t) + 5y(t) = 0; \quad y(1) = -1,\ y'(1) = 13$$