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**Instructions**

1. Write your *Name* and *PID* on the front of your Blue Book.
  2. No calculators or other electronic devices are allowed during this exam.
  3. You may use a double sided page of notes.
  4. Write your solutions clearly in your Blue Book.
    - (a) Carefully indicate the number and letter of each question and question part.
    - (b) Present your answers in the same order as they appear in the exam.
    - (c) Start each numbered problem on a new side of a page.
  5. Show all of your work and justify all your claims. No credit will be given for unsupported answers, even if correct.
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**Complete 10 out of the 11 questions**

1. (10 points) Find the general solution for the system:

$$\mathbf{x}' = \begin{pmatrix} 1 & 1 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 2 \end{pmatrix} \mathbf{x} + \begin{pmatrix} -t \\ 4 - 3t \\ 1 - 2t \end{pmatrix}$$

2. (10 points) Solve the initial value problem:

$$\mathbf{x}' = \begin{pmatrix} 0 & 1 \\ -2 & 3 \end{pmatrix} \mathbf{x}, \quad \mathbf{x}(0) = \begin{pmatrix} 1 \\ -1 \end{pmatrix}$$

3. (10 points) Consider the following differential equation

$$y''(t) - 3y'(t) + 2y(t) = f(t)$$

Using the method of undetermined coefficients, determine the general form of a particular solution  $y_p(t)$  in the following cases (**do not calculate the unknown constants**):

- (a)  $f(t) = t^2 + 1$
- (b)  $f(t) = te^t + t$
- (c)  $f(t) = \sin(t) + \cos(2t)$
- (d)  $f(t) = \sin(t)e^{2t}$

4. (10 points) Solve the equation

$$(y^3 + 4e^x y)dx + (4e^x + 3y^2 x)dy = 0$$

5. (10 points) Solve the initial value problem:

$$\frac{dy}{dx} - \frac{y}{x} = xe^x, \quad y(1) = e - 1$$

6. (10 points) Find the general solution to the following equations:

(a)

$$\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + 10y = 0$$

(b)

$$\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + y = 0$$

7. (10 points) Find the general solution to the differential equation

$$y'' = 5x^{-1}y' - 13x^{-2}y, \quad x > 0$$

How would your answer change if we wanted a solution valid for  $x < 0$ ?

8. (10 points) Using variation of parameters, find a particular solution to the differential equation

$$y'' - 2y' + y = \frac{e^t}{t}$$

9. (10 points) Find a general solution to the system of differential equations

$$\begin{aligned}\frac{dx}{dt} &= x(t) - 4y(t) \\ \frac{dy}{dt} &= x(t) + y(t)\end{aligned}$$

10. (10 points) Solve the initial value problem

$$\frac{dy}{dx} - (1 + y^2)\tan(x) = 0, \quad y(0) = \sqrt{3}$$

11. (10 points) (a) Verify that  $\left\{ \begin{pmatrix} e^{2t} \\ -e^{2t} \end{pmatrix}, \begin{pmatrix} e^{3t} \\ -2e^{3t} \end{pmatrix} \right\}$  is a fundamental solution set to the system

$$\mathbf{x}' = \begin{pmatrix} 1 & -1 \\ 2 & 4 \end{pmatrix} \mathbf{x}$$

(b) Solve the initial value problem

$$\mathbf{x}' = \begin{pmatrix} 1 & -1 \\ 2 & 4 \end{pmatrix} \mathbf{x}, \quad \mathbf{x}(0) = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$