Complete 10 out of the 11 questions

1. (10 points) Find the general solution for the system:

\[ \mathbf{x}' = \begin{pmatrix} 1 & 1 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 2 \end{pmatrix} \mathbf{x} + \begin{pmatrix} -t \\ 4 - 3t \\ 1 - 2t \end{pmatrix} \]

2. (10 points) Solve the initial value problem:

\[ \mathbf{x}' = \begin{pmatrix} 0 & 1 \\ -2 & 3 \end{pmatrix} \mathbf{x}, \quad \mathbf{x}(0) = \begin{pmatrix} 1 \\ -1 \end{pmatrix} \]

3. (10 points) Consider the following differential equation

\[ y''(t) - 3y'(t) + 2y(t) = f(t) \]

Using the method of undetermined coefficients, determine the general form of a particular solution \( y_p(t) \) in the following cases (do not calculate the unknown constants):

(a) \( f(t) = t^2 + 1 \)
(b) \( f(t) = te^t + t \)
(c) \( f(t) = \sin(t) + \cos(2t) \)
(d) \( f(t) = \sin(t)e^{2t} \)

4. (10 points) Solve the equation

\[ (y^3 + 4e^x y)dx + (4e^x + 3y^2 x)dy = 0 \]

5. (10 points) Solve the initial value problem:

\[ \frac{dy}{dx} - \frac{y}{x} = xe^x, \quad y(1) = e - 1 \]

6. (10 points) Find the general solution to the following equations:
(a) \[
\frac{d^2y}{dx^2} + 2 \frac{dy}{dx} + 10y = 0
\]

(b) \[
\frac{d^2y}{dx^2} + 2 \frac{dy}{dx} + y = 0
\]

7. (10 points) Find the general solution to the differential equation

\[y'' = 5x^{-1}y' - 13x^{-2}y, \quad x > 0\]

How would your answer change if we wanted a solution valid for \(x < 0\)?

8. (10 points) Using variation of parameters, find a particular solution to the differential equation

\[y'' - 2y' + y = \frac{\text{e}^t}{t}\]

9. (10 points) Find a general solution to the system of differential equations

\[
\begin{align*}
\frac{dx}{dt} &= x(t) - 4y(t) \\
\frac{dy}{dt} &= x(t) + y(t)
\end{align*}
\]

10. (10 points) Solve the initial value problem

\[
\frac{dy}{dx} - (1 + y^2) \tan(x) = 0, \quad y(0) = \sqrt{3}
\]

11. (10 points) (a) Verify that \(\left\{\left(\text{e}^{2t}, -\text{e}^{2t}\right), \left(\text{e}^{3t}, -2\text{e}^{3t}\right)\right\}\) is a fundamental solution set to the system

\[\mathbf{x}' = \begin{pmatrix} 1 & -1 \\ 2 & 4 \end{pmatrix} \mathbf{x}\]

(b) Solve the initial value problem

\[\mathbf{x}' = \begin{pmatrix} 1 & -1 \\ 2 & 4 \end{pmatrix} \mathbf{x}, \quad \mathbf{x}(0) = \begin{pmatrix} 0 \\ 1 \end{pmatrix}\]