
1. Let \( \phi \) be a function such that \( \frac{\partial \phi}{\partial x}(0, 0, 0) = 2, \frac{\partial \phi}{\partial y}(0, 0, 0) = 3 \) and \( \frac{\partial \phi}{\partial z}(0, 0, 0) = 4 \).
   (a) Let \( w(t) = \phi(c(t)) \), where \( c(t) = t \, i + t^2 \, j + 3t \, k \) is a curve. Find \( \frac{dw}{dt}(0) \)!
   (b) In which direction is the rate of increase of \( \phi \) largest at the point \( (0, 0, 0) \)?
   (c) Let \( \mathbf{F} = \text{grad} \, \phi \). Find \( \text{curl} \, \mathbf{F} \).

2. Let \( f(x, y) = x \cos(x + y) \)
   (a) Calculate the second order Taylor polynomial of \( f \) about the point \( (1, -1) \).
   (b) Use your answer to (a) to write down an estimate for \( f(1.1, -0.8) \).
   (c) Use the linear approximation to find an estimate for \( f(1.1, -0.8) \).

3. Let \( \mathbf{G} = -yi + xj \) be a vector field.
   (a) Show that the curves \( c(t) = r \cos t \, i + r \sin t \, j \), where \( r \) is a constant, are flow lines for \( \mathbf{G} \).
   (b) Sketch the vector field \( \mathbf{G} \) at the points \( (1,0), (0,1), (-1,0) \) and \( (0,-1) \) and sketch the flow line passing through \( (1,0) \).

4. Let \( \mathbf{F}(x, y, z) = (y^2 + x) \, i - (x^2 - y) \, j + z \, k \).
   (a) Find \( \text{curl} \, \mathbf{F} \).
   (b) Find \( \text{div} \, \mathbf{F} \).
   (c) Find the derivative matrix \( \mathbf{D} \mathbf{F} \) (i.e. the matrix of partial derivatives).