Checklist of topics to review

November 6, 2019

Page references refer to Vector Calculus (6th Ed.) - Marsden and Tromba

Chapter 1 - The Geometry of Euclidean Space

§1.1 - Vectors in Two- and Three-Dimensional Space

☐ Vectors
  ☐ Basic definitions and terminology for vectors - (pg 1,2)
  ☐ Basic vector calculations (addition and scalar multiplication) - (pg 2,3)
  ☐ Geometry of vector operations - (pg 4-7)
  ☐ Standard basis vectors - (pg 9)
  ☐ Vector between 2 points (i.e. displacement vector) - (pg 10)

☐ Equations of lines

Typical problems

Finding the equation (in either vector or component form) of the...

☐ Line passing through given point and parallel to given direction
☐ Line between 2 points
☐ Line passing through given point and perpendicular to given plane
☐ Line of intersection between 2 planes
☐ Point of intersection of 2 lines (if any)
☐ Point of intersection of line and plane
☐ Tangent line to a path

☐ Line segments - (pg 16)

☐ Describing points in a Parallelogram with 2 vectors - example 17 (page 16)
§1.2 - The Inner Product, Length, and Distance

- Inner product
  - Definition from components - (page 20)
  - Algebraic properties (one of the important examples: $\mathbf{u} \cdot \mathbf{v} = \mathbf{v} \cdot \mathbf{u}$) - (page 20)
- Length/norm of a vector - (page 20,21)
  - Distance between 2 points - (page 21,22)
- Unit vectors - (page 21)
  - Example 2 (Normalizing vectors) - (page 21)
- Angle between 2 vectors (Theorem 1) - (page 22)
  - Test for perpendicular vectors ($\mathbf{a} \cdot \mathbf{b} = 0$)
- Orthogonal projection - (page 25,26)
  - Calculation using components - example 8
  - Geometric interpretation
- Physical interpretation of vectors (good for developing intuition)
  - Displacements points in space - example 10 (page 27)
  - Velocity vector (and connection to displacement) - example 11 (page 28)
  - Resultant forces - example 12 (page 29)

§1.3 - Matrices, Determinants, and the Cross Product

- Matrices
  - Basic definitions and terminology - (page 31)
  - Calculations (sums of matrices, products of matrices, algebraic rules)
- Determinant calculations
  - For $2 \times 2$ matrix - (page 31)
  - For $3 \times 3$ matrix - (page 32)
- Determinant properties (can be useful for $3 \times 3$ matrices) - (page 32,33)
- Cross product
  - Calculation from components ($3 \times 3$ determinant formula) - (page 35)
  - Geometric definition (right hand rule) - (page 37)
□ Algebraic rules (one of the important examples: \( \mathbf{u} \times \mathbf{v} = -\mathbf{v} \times \mathbf{u} \)) - (page 37)

□ Calculating area of a parallelogram

□ In \( \mathbb{R}^2 \) - (page 39)

□ In \( \mathbb{R}^3 \) - (page 37, 38)

□ Calculating volume of parallelepiped - (page 40, 41)

□ Equations of planes

Typical problems

Finding the equation of the...

□ Plane perpendicular to a given vector and passing through a point

□ Plane through 3 points

□ Plane through a given point and perpendicular to a given line

□ Plane containing a line and a point (not on the line)

□ Plane containing two non-parallel lines

□ Distance from plane to point - (page 43, 44)
Chapter 2 - Differentiation

§2.1 - The Geometry of Real-Valued Functions

- Function notation and terminology - (page 76,77)
- Graphs of functions - (page 77)
- Level sets
  - Level curves (for 2 variables) - (page 78)
  - Level surfaces (for 3 variables) - (page 79)

§2.3 - Differentiation

- Partial derivatives
  - Limit Definition - (page 106)
  - Calculation - example 1 (page 106, 107)
  - Geometric interpretation (for functions of 2 variables)
  - Affine approximation - (page 108)
- Tangent plane (to a graph \( z = f(x,y) \)) - (page 110)
- Matrix of partial derivatives - (page 111,112)
- Gradient (vector field)\(^1\) \( \nabla f \) - (page 112,113)

- Basic theorems
  - Theorem 8 [Differentiable at a point \( \Rightarrow \) continuous at a point] - (page 113)
  - Theorem 9 [\( f \) differentiable at a point \( \Leftrightarrow \) partial derivatives all exist and are continuous at that point] - (page 113)
- Non-differentiable functions (in 2 variables) - example 10 (page 114)

§2.4 - Introduction to Paths and Curves

- Paths
  - Basic notation and terminology - (page 116,117)
- Problems involving paths:
  - Finding points on a path
  - Point of intersection between curve (parametrized by path) and surface

\(^1\)The gradient is a very important function and so will be the focus of §2.6
\[\square\text{Intersecting paths}\]
\[\square\text{Basic examples of paths in the plane } (\mathbb{R}^2) \] - (page 117-119)
\[\quad \square \text{Line in space - example 1}\]
\[\quad \square \text{Tracing circle in the plane - example 2}\]
\[\quad \square \text{Parabola - example 3}\]
\[\quad \square \text{Cycloid}^2 \text{ example 4}\]
\[\square \text{Basic examples of paths in space } (\mathbb{R}^3) \] - (page 121-123)
\[\quad \square \text{Example 5}\]
\[\quad \square \text{Helix - example 6}\]
\[\quad \square \text{Particle in space - example 9}\]
\[\square \text{Velocity of a path - (page 120)}\]
\[\square \text{Vector tangent to a path (at a point) - (page 120)}\]
\[\quad \square \text{Tangent line to a curve - (page 122)}\]

\[\textbf{\textsection 2.5 - Properties of the Derivative}\]
\[\square \text{Sums, Products, Quotients (Theorem 10) - (page 125)}\]
\[\square \text{Chain rule (Theorem 11) - (page 126)}\]
\[\quad \square \text{Calculation using matrix products}\]
\[\square \text{Special case of chain rule } (p = f \circ c) \] - (page 127)
\[\quad \square \text{Geometric interpretation for the chain rule (and matrix of partial derivatives) - (page 129, 130)}\]

\[\textbf{\textsection 2.6 - Gradients and Directional Derivatives}\]
\[\square \text{Directional derivatives - (page 135, 136)}\]
\[\quad \square \text{Definition}\]
\[\quad \square \text{Calculation (using gradients)}\]
\[\quad \square \text{Geometric interpretation (rate of change/slope of a function at a point in a given direction)}\]
\[\square \text{Direction of fastest increase of a function (Theorem 13) - (page 137, 138)}\]
\[\quad \square \text{Finding direction of greatest rate of change - example 5 (page 138)}\]
\[\square \text{The gradient of normal to level surfaces (Theorem 14) - (page 138)}\]

\[^2\text{This is an interesting example...}\]
☐ Finding normal direction to a surface (and more generally level set of a function)

☐ Tangent plane (to a level surface $f(x, y, z) = k$) - (page 139)
Chapter 5 - Double and Triple Integrals

§5.1 - Introduction
- Volume definition of double integral - (page 263, 264)
- Basic examples - that do not require integration (example 1 - constant functions, triangular prisms) - (page 264)
- Reducing double integrals (over rectangular R) to iterated integrals - (page 267)
- Geometric interpretation (Cavalieri’s Principle) - (page 265-267)

§5.2 - The Double Integral over a Rectangle
- Limit definition of double integral - (page 272)
- Basic properties of double integrals (linearity, homogeneity, monotonicity, additivity) - (page 275)
- Fubini’s Theorem (Theorem 3) - (page 277)
  - Application to problem solving: Order of integration over rectangles does not matter - examples 1, 2, 3 (page 279-280)

§5.3 - The Double Integral Over More General Regions
- Elementary regions - (page 283)
- Rewriting a region in \( \mathbb{R}^2 \):
  - As \( y \)-simple
  - As \( x \)-simple
- Reducing double integrals (over general \( R \)) to iterated integrals (Theorem 4 and 4') (page 286, 287)

§5.4 - Changing the Order of Integration
- Rewriting \( y \)-simple regions as \( x \)-simple (and vice versa)
- Changing order of integration - (page 289-290)
- Evaluating difficult (or impossible) integrals by changing order of integration - examples 1 and 2 (page 290-291)
  - Example 1 - (page 290)
    
    \[ f(x, y) = (a^2 - y^2)^{1/2} \]
    
    (Easier to integrate wrt \( x \) first)
\[ f(x, y) = (x - 1)\sqrt{1 + e^{2y}} \]

(Easier to integrate wrt \( x \) first)

\[ f(x, y) = xe^{y^3} \]

(Easier to integrate wrt \( x \) first)

\[ f(x, y) = \sin(y^3) \]

(Easier to integrate wrt \( x \) first)

§5.5 - Triple Integrals

- Definition of a triple integral (limiting case of an approximation) - (page 294-295)

- Reduction of a triple integral to iterated integrals over a box \( B \) - (page 295-296)
  - Example 1 - (page 296)
  - Example 2 - (page 297)

- Elementary regions in space - (page 297)

- Rewriting regions in \( \mathbb{R}^3 \) as an elementary region
  - Example 3 - (page 297, 298)
  - Example 5 - (page 300, 301)

- Reduction of a triple integral to iterated integrals over an elementary region - (page 298)
  - Example 4 - (page 298, 299)
  - Example 5 - (page 300, 301)

- Finding volume of a solid (by integrating 1)
  - Example 4 - (page 298, 299)