1. Write your Name and PID on the front of your Blue Book.
2. No calculators or other electronic devices are allowed during this exam.
3. You may use a double sided page of notes.
4. Write your solutions clearly in your Blue Book.
   (a) Carefully indicate the number and letter of each question and question part.
   (b) Present your answers in the same order as they appear in the exam.
   (c) Start each numbered problem on a new side of a page.
5. Show all of your work and justify all your claims. No credit will be given for unsupported answers, even if correct.

1. (10 points) Solve the following integral equation

   \[ y(t) + \int_0^t e^{t-\nu} y(\nu) d\nu = \sin(t) \]

2. (10 points) Determine the correct form for a particular solution to the following non-homogeneous differential equations
   (a) \( y'' - 2y' + 2y = te^t \cos(t) \)
   (b) \( y'' = t^2 - t + 1 \)
   (c) \( y'' - y = \cos(t) - \sin(t) + \sin(2t) \)
   (You do not need to calculate coefficients)

3. (10 points) Solve the symbolic initial value problem

   \[ \frac{d^2x}{dt^2} + 4x = 6\delta(t - \pi); \quad x(0) = 2, \quad \frac{dx}{dt}(0) = 0 \]

4. (10 points) By finding the generalized eigenvalues or otherwise, find the general solution to the homogeneous system

   \[ x' = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{pmatrix} x \]

5. (10 points) Suppose \( y(t) \) is a solution to the initial value problem

   \[ ty'' + 2(t - 1)y' - 2y = 0; \quad y(0) = 0, y'(0) = 0 \]

   Find the Laplace transform of \( y(t) \)

6. (10 points) Consider the following discontinuous function

   \[ g(t) = \begin{cases} 
   0 & 0 \leq t < 1 \\
   t^2 & 1 \leq t < 2 \\
   t^2 + t - 2 & t \geq 2 
   \end{cases} \]
(a) Express \( g(t) \) in terms of unit step functions \( u(t) \)

(b) Find the Laplace transform of \( g(t) \)

7. (10 points) Find a general solution to the following non-homogeneous system of differential equations

\[
\begin{align*}
\frac{dz_1}{dt}(t) &= 6z_1(t) + z_2(t) - 11 \\
\frac{dz_2}{dt}(t) &= 4z_1(t) + 3z_2(t) - 5
\end{align*}
\]

8. (10 points) Solve the initial value problem

\[
\frac{dy}{dx} = \frac{y - 2x}{2y - x}, \quad y(1) = 1
\]

9. (10 points) (a) Verify that \( \left\{ \begin{pmatrix} e^{7t} \\ 2e^{7t} \end{pmatrix}, \begin{pmatrix} e^{-5t} \\ -2e^{-5t} \end{pmatrix} \right\} \) is a fundamental solution set to the system

\[
x' = \begin{pmatrix} 1 & 3 \\ 12 & 1 \end{pmatrix} x
\]

(b) Solve the initial value problem

\[
x' = \begin{pmatrix} 1 & 3 \\ 12 & 1 \end{pmatrix} x, \quad x(0) = \begin{pmatrix} 4 \\ 0 \end{pmatrix}
\]

(c) Given a system of differential equations \( x' = Ax \) with fundamental matrix \( X(t) \), the matrix exponential function \( e^{At} \) satisfies

\[
e^{At} = X(t)X(0)^{-1}
\]

where \( X(0)^{-1} \) denotes the inverse matrix to \( X(0) \). Using part (a) or otherwise, calculate \( e^{At} \)

10. (10 points) Find a particular solution to the differential equation

\[
\frac{d^2y}{dt^2} - 2\frac{dy}{dt} + y = e^t
\]

11. (10 points) Find an explicit solution to the initial value problem

\[
e^{2y} \frac{dy}{dx} = 8x^3, \quad y(1) = 0
\]

For which values of \( x \) is this solution valid?