Instructions

1. Write your Name and PID on the front of your Blue Book.
2. No calculators or other electronic devices are allowed during this exam.
3. You may use a double sided page of notes.
4. Write your solutions clearly in your Blue Book.
   (a) Carefully indicate the number and letter of each question and question part.
   (b) Present your answers in the same order as they appear in the exam.
      (c) Start each numbered problem on a new side of a page.
5. Show all of your work and justify all your claims. No credit will be given for unsupported answers, even if correct.

Complete 10 out of the 11 questions

1. (10 points) Find the general solution for the system:

\[
x' = \begin{pmatrix} 1 & 1 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 2 \end{pmatrix} x + \begin{pmatrix} -t \\ 4 - 3t \\ 1 - 2t \end{pmatrix}
\]

2. (10 points) Solve the initial value problem:

\[x' = \begin{pmatrix} 0 & 1 \\ -2 & 3 \end{pmatrix} x, \quad x(0) = \begin{pmatrix} 1 \\ -1 \end{pmatrix}\]

3. (10 points) Consider the following differential equation

\[y''(t) - 3y'(t) + 2y(t) = f(t)\]

Using the method of undetermined coefficients, determine the general form of a particular solution \(y_p(t)\) in the following cases (do not calculate the unknown constants):

(a) \(f(t) = t^2 + 1\)
(b) \(f(t) = te^t + t\)
(c) \(f(t) = \sin(t) + \cos(2t)\)
(d) \(f(t) = \sin(t)e^{2t}\)

4. (10 points) Solve the equation

\[(y^3 + 4e^x y)dx + (4e^x + 3y^2 x)dy = 0\]

5. (10 points) Solve the initial value problem:

\[\frac{dy}{dx} - \frac{y}{x} = xe^x, \quad y(1) = e - 1\]

6. (10 points) Find the general solution to the following equations:
(a) \[
\frac{d^2 y}{dx^2} + 2 \frac{dy}{dx} + 10y = 0
\]

(b) \[
\frac{d^2 y}{dx^2} + 2 \frac{dy}{dx} + y = 0
\]

7. (10 points) Find the general solution to the differential equation
\[
y'' = 5x^{-1}y' - 13x^{-2}y, \quad x > 0
\]

How would your answer change if we wanted a solution valid for \(x < 0\)?

8. (10 points) Using variation of parameters, find a particular solution to the differential equation
\[
y'' - 2y' + y = \frac{e^t}{t}
\]

9. (10 points) Find a general solution to the system of differential equations
\[
\begin{align*}
\frac{dx}{dt} &= x(t) - 4y(t) \\
\frac{dy}{dt} &= x(t) + y(t)
\end{align*}
\]

10. (10 points) Solve the initial value problem
\[
\frac{dy}{dx} - (1 + y^2) \tan(x) = 0, \quad y(0) = \sqrt{3}
\]

11. (10 points) (a) Verify that \(\left\{\left(\begin{array}{c} e^{2t} \\ -e^{2t} \end{array}\right), \left(\begin{array}{c} e^{3t} \\ -2e^{3t} \end{array}\right)\right\}\) is a fundamental solution set to the system
\[
x' = \begin{pmatrix} 1 & -1 \\ 2 & 4 \end{pmatrix} x
\]

(b) Solve the initial value problem
\[
x' = \begin{pmatrix} 1 & -1 \\ 2 & 4 \end{pmatrix} x, \quad x(0) = \begin{pmatrix} 0 \\ 0 \end{pmatrix}
\]