

identification. Unfortunately it does not cover the later work on internal code assignments to the states, the decomposition theory of finite automata and its beautiful connection to the semigroup of the machine. Clearly, there has been much more work on finite automata which a 1963 book cannot cover but which could be well included in a modern text; for example, linear automata, shift-register realizations of automata, one-binary-feedback loop realizations (it should be observed that this nice result is even missing from some very recent texts) and possibly probabilistic automata.

The treatment of algorithms and Turing machines is elementary but again well written if somewhat lengthy. Here it is strange to note that the authors do not prove that there exist noncomputable functions and undecidable problems. Undecidable problems are mentioned (see Trakhtenbrot's theorem, etc.), but for proofs one is advised to read Davis. After much of the necessary background is developed it is startling that a simple proof of the undecidability of the "halting problem" is not included.

A look at the Bibliography is quite disappointing since even with a Supplement to Bibliography and an Addenda to Bibliography the listing barely reaches 1965. Furthermore, it appears that the translator has used the original numbering of references. This saves the translator some work but it mixes up partially the alphabetic order and some warning or help should have been given to the unsuspecting reader. Even so, one wonders why, Shannon appears as reference number 110 and 229, 230 and 231. A look at the index is no help at all since it does not contain Shannon, regardless of the fact that Shannon is explicitly mentioned on p. 1 (in a charmingly chauvinistic way). One also notes that the back-and-forth translation of Gill's name has tripped up the translator: the Hill's method of p. 401 appears to be the same as Gill's method on p. 250.

Finally, the left-hand page headings all through the book keep announcing that one is reading *Elements of Mathematical Logic*, which is only the title of the first chapter. Somebody at Academic Press was in too much of a hurry. This was a very good book in 1963 but even in 1971, when a translation is hard to justify, it should have been done more carefully.

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Principles of Combinatorics. By C. BERGE. Academic Press, New York, 1971. viii + 176 pp. \$10.00.

In recent years, books in the general area which is becoming known as combinatorial theory have been appearing at a significantly increasing rate. A welcome addition to this growing list is *Principles of Combinatorics* by Claude Berge. A faithful translation of his *Principes des combinatoires* which appeared in 1968, it is based on a series of introductory lectures delivered by Berge in the Faculté des Sciences de Paris in 1967–68. In the words of the author, "the present work is concerned only with the problem of counting, that is, the oldest pre-occupation of combinatorics." Do not be misled by this however, since the approach taken by Berge is thoroughly modern in its viewpoint. Indeed, defining a *configuration* as a *mapping of a set of objects into a finite abstract set with a given structure*, Berge, in his thoughtful introduction, interprets combinatorics as that field which

“... counts, enumerates, examines, ... and investigates the existence of configurations with certain specified properties, ... , looks for their intrinsic properties, and studies transformations of one configuration into another as well as ‘subconfigurations’ of a given configuration.”

The book is divided into five chapters, roughly equal in length.

In Chap. 1 the reader is first introduced to a number of definitions which will be essential for the rest of the book. Berge then proceeds to derive most of the basic functions encountered in elementary counting theory, pausing occasionally along the way to show applications of the ideas, e.g., Seidenberg’s elegant proof of the Erdős–Szekerés monotone subsequence theorem and Rota’s short derivation of the exponential generating function for Bell numbers. A major theme running through this chapter is the idea of mapping. The variations on this theme serve to unify the various quantities under consideration. For example, if $|X| = n$, $|A| = m$, then (i) the number of mappings of X into A is m^n ; (ii) the number of injections of X into A is $[m]_n$ (the falling factorial); (iii) the number of “increasing” mappings of X into A is equal to $[m]^n/n!$ (rising factorial in numerator); (iv) the number of surjections of X into A is $m! S_n^m$ (S_n^m is a Stirling number of the second kind); etc.

Chapter 2 deals with combinatorial aspects of the theory of partitions. Specifically, by looking carefully at the Ferrers diagram (also known as the Ferrers graph) of a partition, the author derives a number of classical results; e.g., the number of self-conjugate partitions of n equals the number of partitions of n with all parts unequal and odd, and the number of partitions of n into distinct parts equals the number of partitions of n into odd parts. A nice proof of this type is also given for the Euler pentagonal number theorem. The second half of the chapter is concerned with tableaux associated with partitions and includes several nice results of G. de B. Robinson and C. Shensted.

The topic of Chap. 3 is inversion formulas and their applications. Starting with the concept of a differential operator associated with a normal family of polynomials, Berge immediately develops several “binomial-type” theorems, e.g.,

$$[x + y]_n = \sum_k \binom{n}{k} [x]_k [y]_{n-k}, [x + y]^n = \sum_k \binom{n}{k} [x]^k [y]^{n-k}, \text{ and their inverses, e.g.,}$$

$$S_m^n = (1/m!) \sum_k (-1)^{m-k} \binom{m}{k} k^n. \text{ He then presents Rota’s development of the}$$

Möbius function for locally-finite partially-ordered sets. The reader should find the examples given here especially appropriate. Of course, an immediate application is to the principle of inclusion–exclusion which, though not mentioned by this name, is embedded in a section entitled “Sieve formulas.” The chapter concludes with a comprehensive summary of results in tree enumeration, the simplest, of course, being Cayley’s n^{n-2} theorem.

Chapter 4 runs in a somewhat different vein from the other chapters. It deals with elementary aspects of permutation groups and is the most algebraic of the five chapters. Presumably, it is intended as background for the Polyá theory to follow in the next chapter. However, Berge does strike a somewhat broader course, e.g., showing that A_n is the only normal subgroup of S_n for $n > 4$ (which, incidentally, prompts the statement of a rather original variation of Galois’ theorem

on solvability of algebraic equations by radicals (p. 135)), introducing the permutohedron of Guilbaud and Rosenstiehl, and presenting several pleasant theorems relating transpositions, induced diagrams and the symmetric group.

The final chapter is accurately entitled "Polyá's Theorem." Using a lemma of Burnside from the preceding chapter, the well-known theorem of Polyá is established. The development is clear though brief and the examples again are well-chosen. The chapter concludes with an application to knot-counting and a generalization of the Polyá theorem due to de Bruijn.

The text is quite readable although occasional inconsistencies crop up, e.g., defining Abelian groups (p. 29) but using vector space and homomorphism without definitions, and defining Fibonacci numbers by $F_n = \sum_k \binom{n-k+1}{k}$ but never mentioning the recurrence $F_{n+2} = F_{n+1} + F_n$. The reviewer also felt that the inclusion of exercises would have aided the potential reader in testing his grasp of the material although the examples and applications included certainly help alleviate this deficiency.

In summary, Berge has given us a book which may, as Gian-Carlo Rota says in his enthusiastic Foreword, help persuade those who would "unknot themselves from the tentacles of the Continuum and join the Rebel Army of the Discrete."

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Nonparametric Statistical Inference. By JEAN DICKINSON GIBBONS. McGraw-Hill, New York, 1971. xiv + 306 pp. \$11.95.

In the preface the author describes the level of the book:

The texts presently available which are devoted exclusively to nonparametric statistics are few in number and seem to be predominantly either of the handbook style, with few or no justifications, or of the highly rigorous mathematical style. The present book is an attempt to bridge the gap between these extremes. It assumes the reader is well acquainted with statistical inference for the traditional parametric-estimation and hypothesis-testing procedures, basic probability theory and random-sampling distributions. The survey is not intended to be exhaustive, as the field is so extensive. The purpose of the book is to provide a compendium of some of the better-known nonparametric techniques for each problem situation. Those derivations, proofs, and mathematical details which are relatively easily grasped or which illustrate typical procedures in general nonparametric statistics are included. More advanced results are simply stated with references.

This is indeed a very apt description.

The scope of the book is well indicated by the following chapter headings: 1. Introduction, review, and notation; 2. Order statistics; 3. Tests based on runs; 4. Tests of goodness of fit; 5. Rank-order statistics; 6. Other one-sample and paired-sample techniques: The sign test and signed-rank test; 7. The general two-sample problem; 8. Linear rank statistics and the general two-sample problem; 9. Linear rank tests for the location problem; 10. Linear rank tests for the scale problem; 11. Tests of the equality of k independent samples; 12. Measures of association for bivariate samples; 13. Measures of association in multiple classifications; 14. Asymptotic relative efficiency. The reviewer would disagree that this is a