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MYSTERY TILES AT MURRAY HILL

An earlier tale, reprinted as Puzzle 15 in my *Science Fiction Puzzle Tales* (1981) reported on the work of French archeologists of the 25th century in unearthing the ruins of what had once been New Jersey. The state had been totally destroyed by the great nuclear war of 2037. The same group, after completing its Secaucus dig, turned its attention to Murray Hill in the hope of finding some remnants of Bell Laboratories.

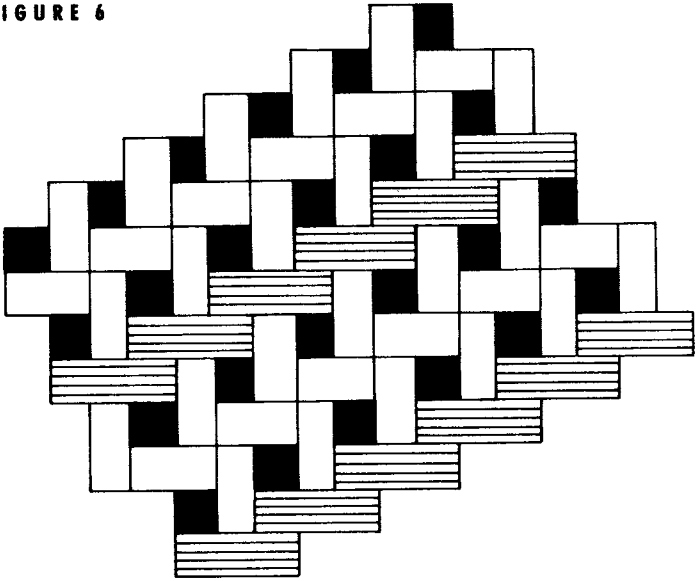
Their efforts were soon rewarded. Large portions of the lab's main building were found intact. One small office, on a floor where Bell's mathematicians worked, had a wall covered with a curious periodic pattern of brightly colored ceramic tiles. Figure 6 shows what a fragment of the wall looked like.

Back in Paris, a group of French mathematicians studied the pattern. "It looks like a conventional tiling with three kinds of straight polyominoes," said Clyde Barge, a noted graph theorist.

"I can't believe it's not more than that," said Doris Snapshutter, Clyde's associate. "After all, we know the office belonged to Ronald L. Graham, the famous combinatorialist who discovered the Graham tile."

The Graham tile, found in 1986, is the only known shape that tiles the plane only in a nonperiodic way. It is a nonconvex polygon with 71 sides. Graham was perhaps better known to the general public in his day for his work with Marvin Minsky in constructing the first robot capable of juggling 100 balls.

FIGURE 6

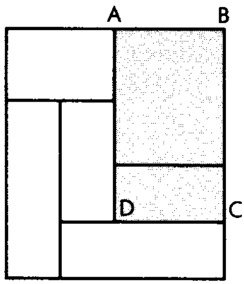


"I see what you mean" said Clyde. "It's unlikely a man of Graham's interests would have had his wall tiled in anything but some sort of remarkable pattern. See if you can find something about it in the literature of the period."

Doris spent several days at her computer console, searching old twentieth-century math journals for papers on polygonal tiling. She finally found what she was looking for. A paper in a 1982 issue of *Mathematics Magazine,* written by Graham and four friends (Fan Chung, E. N. Gilbert, J. B. Shearer, and J. H. van Lint), described the pattern.

The five mathematicians had set themselves the following unusual task. A rectangle is divided into smaller rectangles, all sides of which are integral, and in such a way that no sub-rectangle is formed by two or more rectangles. Such a tiling is said to be "irreducible." For example, the

FIGURE 7

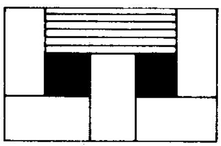


pattern in Figure 7 is not irreducible because of the subrectangle *ABCD.*

The problem is this. What is the smallest average area of tiles that can be obtained by tiling a rectangle according to the rules? Of course without the proviso about irreducibility, the minimum average is 1. You simply divide the rectangle into unit squares.

Figure 8 shows how a rectangle can be divided to obtain 1.875 as the average area of a tile. (The average is the total area of 15, divided by the number of tiles, 8.) Can this average be lowered?

FIGURE 8



It can. In 1980 Graham and his colleagues discovered the infinite pattern that Graham had placed on his wall. By a clever use of what are called "weighting functions," they were able to show that this is the only irreducible pattern of integral rectangles that tiles the entire plane in such a way that the average area of a tile is minimized.

"But can a rectangle be tiled so as to achieve this minimum?" Clyde asked.

"No," replied Doris. "But it can come arbitrarily close. You simply take a very large portion of the infinite pattern, as nearly rectangular as possible, then fill it in around the ragged edges. By going to larger and larger hunks of the pattern, you can reduce the average area of a tile to a value as close to the minimum as you please."

What is this minimum average? It is easily calculated from the pattern.