THE EUREKA MOMENT

RONALD GRAHAM

INTERVIEWED BY JOAN RACHEL GOLDBERG

Intriguing as Ronald Graham's "Guinness Book of World Records" achievement is—creation of the highest number ever used in a mathematical proof—he is best known in mathematics circles for its context: his pioneering work in the field known as Ramsey theory. Ramsey theory deals with questions of chaos and order. It stipulates, for example, that in a group of six or more people chosen at random, either three of them will all know one another or three of them will all be strangers. As the group size increases, however, statements such as these become increasingly difficult to prove mathematically.

For this work, Graham was awarded the Pólya Prize, the premier award in combinatorics, a branch of mathematics that deals with finite sets.

At 49, Graham is director of AT&T Bell Laboratories Mathematical Sciences Research Center, home to about 70 world-class mathematicians. He began his career there 22 years ago, after earning a Ph.D. at the University of California, Berkeley.

Graham teaches computer science at Stanford and Caltech, serves on the editorial boards of 25 mathematical journals and is presently at work on a popular book about magic and mathematics. In addition, he is a consummate juggler and trampolinist.

Here Graham speaks of how varied interests influence his approach to mathematics.

It's not an easy business to know how you think creatively. Some problems are much easier to approach in one form than in another. Harder problems are sometimes easier to solve. Knowing the right question to ask is often half the battle.

Ramsey theory is an area of mathematics that I've had some nice experiences with. It's a branch that deals with structure preserved under partitions. Typically, one looks at the following kind of question: If a particular mathematical structure is arbitrarily partitioned into finitely many classes, what kind of substructures must always remain intact in at least one of the classes?

I had been thinking of a problem in Ramsey theory for some months. And I felt I was getting close to seeing more clearly what was really going on. Often I find if I'm thinking about something right up to going to bed, that isn't as productive as thinking about it and then stopping a few hours before sleep to let my mind cool down a little bit. If my mind's in a more active phase, then it doesn't shut off as rapidly. There are theories and some evidence about this that make sense to me. These suggest that sleeping time is the time to sort out what makes sense from what doesn't, throw away irrelevant things and get your house in order.

Music is very useful in allowing my mind to make this transition, but it must be fairly structured. Much music composed before 1940, and some after, typically has an obvious structure, has certain patterns. There are predictable patterns in juggling, math and computer science as well. If you want to create new juggling tricks, as I do from time to time, then it's almost a mathematical problem.

Joan Rachel Goldberg is the author of the new book High-Tech Career Strategies for Women.

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So I like to listen to music before I go to sleep and allow my thoughts to flow. Mozart, Bach, Beethoven and Brahms are better than John Cage. The structure of the music is intertwined with mathematics. The music plays and my mathematical thoughts flow, and they merge. Music is a nice adjunct to thinking. It helps reinforce patterns that may be developing in the space between waking and sleeping.

I can usually be aware of exactly when it is that I'm falling asleep. In this case, I was in that phase near sleep when things become interrelated or interconnected that wouldn't normally. I can't say that I have many mystical experiences, but in this instance I really had this feeling that I saw the whole field of Ramsey theory, in a sense, laid out in front of me, as though it were an infinite sequence of nested cubes.

In thinking of Ramsey theory, a mathematician tends to see more the outline of a cube than a solid cube. It's

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as if all the edges and corners are joined together, as if they're made out of wire, so that cubes can be visualized as inside one another. These configurations can represent higher-and higher-dimensional cubes. Say you want to represent the next-higher-dimensional cube beyond a square—a 3-D cube—you could draw another square nearby and connect the four corresponding vertices. To represent the next-higher-dimensional cube, you could draw something that looks like a cube within a cube and connect corresponding vertices, and so forth. So the vision I had fit in with my work on Ramsey theory, because that Guinness-world-record number really denotes a number of dimensions.

It helps put things into a certain perspective when you see how things interrelate and fit together, as in the nested cubes. It gives a little more global structure to the whole area you're working on. And there's always an increasing need for that kind of synthesis, because in mathematics, as in a lot of fields, the basic efforts are to push farther out, past the frontiers.

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I saw this vision when I was just falling asleep. When I woke up, it was a matter of somehow writing things down, confirming and looking at special instances. Really, it was just like picking the fruit, so to speak, off this big tree that I had seen in its entirety for the first time. It came together more than at any time before. It was almost as if I could view it from a higher level and see where things fit into place more clearly.

You get the big picture geometrically, and then you carry out a detailed mathematical analysis. Intuition is really a key ingredient in mathematics; a lot of people think it's mechanical, like turning a crank, but in fact it's just the opposite. The brain is working on many different tracks at once. Often the problem is how to weed out the stuff that's irrelevant.

Eventually, I had to get back down, really on the front lines, and start proving the theorem. But that vision helped me form an overall plan of attack.

Young people don't know what's impossible; they're willing to try crazy things. If you're older, you get more conservative. I think it's essential not to be afraid to try crazy things. ■