(I) Write a program that takes data points \((x_i, y_i), \ i = 0, ..., n\), and outputs the Newton form of the interpolating polynomial (suffices to compute the coefficients). Write another program for the Lagrange form. Using the code you wrote, interpolate the function \(f(x) = \sin(2\pi x)\) at ten equi-spaced nodes in \([0,1]\), including 0 and 1. Overlay the value of the interpolating polynomial at twenty equi-spaced points in \([0, 1]\) (including 0 and 1).

Verify that both your programs give the same results.

To be clear, here is what you should submit:

(a) A program for constructing the Newton form of the interpolation polynomial.

(b) A program for constructing the Lagrange form of the interpolation polynomial.

(c) A plot of the interpolation polynomial constructed using 10 equi-spaced nodes and a plot of the interpolation polynomial constructed using 20 equi-spaced nodes. These plots should be made using your program for the Newton form. Submit one graph with two plots on it or two graphs with one plot on each.

(d) A plot of the interpolation polynomial constructed using 10 equi-spaced nodes and a plot of the interpolation polynomial constructed using 20 equi-spaced nodes. These plots should be made using your program for the Lagrange form. Submit one graph with two plots on it or two graphs with one plot on each.

(e) A brief (2-3 sentence) explanation of your results. In particular, how do the graphs in (c) and (d) compare?