Homework 1 for Math 277A - Fall 2018

(I) Given i.i.d. samples $x_1, ..., x_p \in \mathbb{R}^d$, drawn from the same distribtution with mean μ and covariance Σ, show that

  (a) $\mu_p = \frac{1}{p}\sum_{i=1}^{p}$ is an unbiased estimator for μ.

  (b) $\Sigma_p = \frac{1}{p-1}\sum_{i=1}^{p}(x_i - \mu_p)(x_i - \mu_p)^T$ is an unbiased estimator for Σ.

(II) Prove the convolution theorem for the Discrete Fourier Transform, and prove that the DFT matrix diagonalizes any circulant matrix.

(III) Prove that the sequence of subspaces generated by the Haar scaling function forms an MRA. That is, check that the 6 properties of an MRA hold.