1. Consider Newton’s method to optimize the function $F(x) = \frac{x^2}{2} + x - \epsilon \log(x)$ on $\mathbb{R}^+$, where $1 > \epsilon > 0$ is a given constant.

(a) Write down explicitly what the Newton update is.
(b) Suppose $\hat{x}$ is a minimizer of $F$. Show there exists some sequence $\{\beta_k\}$ such that

$$x^{(k+1)} - \hat{x} = \beta_k (x^{(k)} - \hat{x})^2$$

2. Let $B > 0$. Given a constant matrix $A \in \mathbb{R}^{m \times n}$ and $b \in \mathbb{R}^m$, formulate the dual to the following primal problem:

$$\min_x \quad x^T B x$$

s.t. $\quad Ax \leq b$

3. Show that strong duality fails to hold for the following problem with domain $D = \{(x, y) : y > 0\}$:

$$\min_{x,y \in D} \quad e^{-x}$$

s.t. $\quad \frac{x^2}{y} \leq 0$