1. (5 points) Consider the matrix $A = \begin{pmatrix} 3 & 5 \\ 5 & 3 \end{pmatrix}$. Find the eigenvalues and corresponding eigenvectors of $A$. 
2. (10 points) Find a general solution to the system of ODEs given by

\[ \dot{x} = \begin{pmatrix} -6 & 2 \\ -3 & 1 \end{pmatrix} x. \]
3. (10 points) Find a general solution to the system of ODEs given by

\[ \mathbf{x}' = \begin{pmatrix} 1 & -8 \\ 1 & -3 \end{pmatrix} \mathbf{x}. \]
4. (5 points) Let $A$ be a $15 \times 15$ matrix, and suppose $A$ has the following eigenvalues with corresponding algebraic and geometric multiplicities:

<table>
<thead>
<tr>
<th>Eigenvalue $\lambda_i$</th>
<th>Algebraic Multiplicity</th>
<th>Geometric Multiplicity</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\lambda_1 = 2$</td>
<td>3</td>
<td>2</td>
</tr>
<tr>
<td>$\lambda_2 = -1$</td>
<td>6</td>
<td>1</td>
</tr>
<tr>
<td>$\lambda_3 = 8$</td>
<td>5</td>
<td>2</td>
</tr>
<tr>
<td>$\lambda_4 = 3$</td>
<td>1</td>
<td>1</td>
</tr>
</tbody>
</table>

(a) (1 point) What is the degree of the characteristic polynomial of $A$?

(b) (1 point) How many generalized eigenvectors are associated with $\lambda_1$?

(c) (1 point) How many generalized eigenvectors are associated with $\lambda_2$?

(d) (1 point) How many generalized eigenvectors are associated with $\lambda_3$?

(e) (1 point) How many generalized eigenvectors are associated with $\lambda_4$?
5. (10 points) Find a general solution to the system of ODEs

\[ \dot{x} = \begin{pmatrix} 12 & -9 \\ 4 & 0 \end{pmatrix} x. \]
6. (10 points) Consider the system of ODEs given by

\[ x' = \begin{pmatrix} 3 & -3 \\ 2 & -2 \end{pmatrix} x + \begin{pmatrix} 4 \\ -1 \end{pmatrix}. \]

Using that \( x_1 = \begin{pmatrix} 1 \\ 1 \end{pmatrix} \) and \( x_2 = \begin{pmatrix} 3e^t \\ 2e^t \end{pmatrix} \) are linearly independent solutions to the associated homogeneous system, use the method of variation of parameters to find a general solution to the original nonhomogeneous system.